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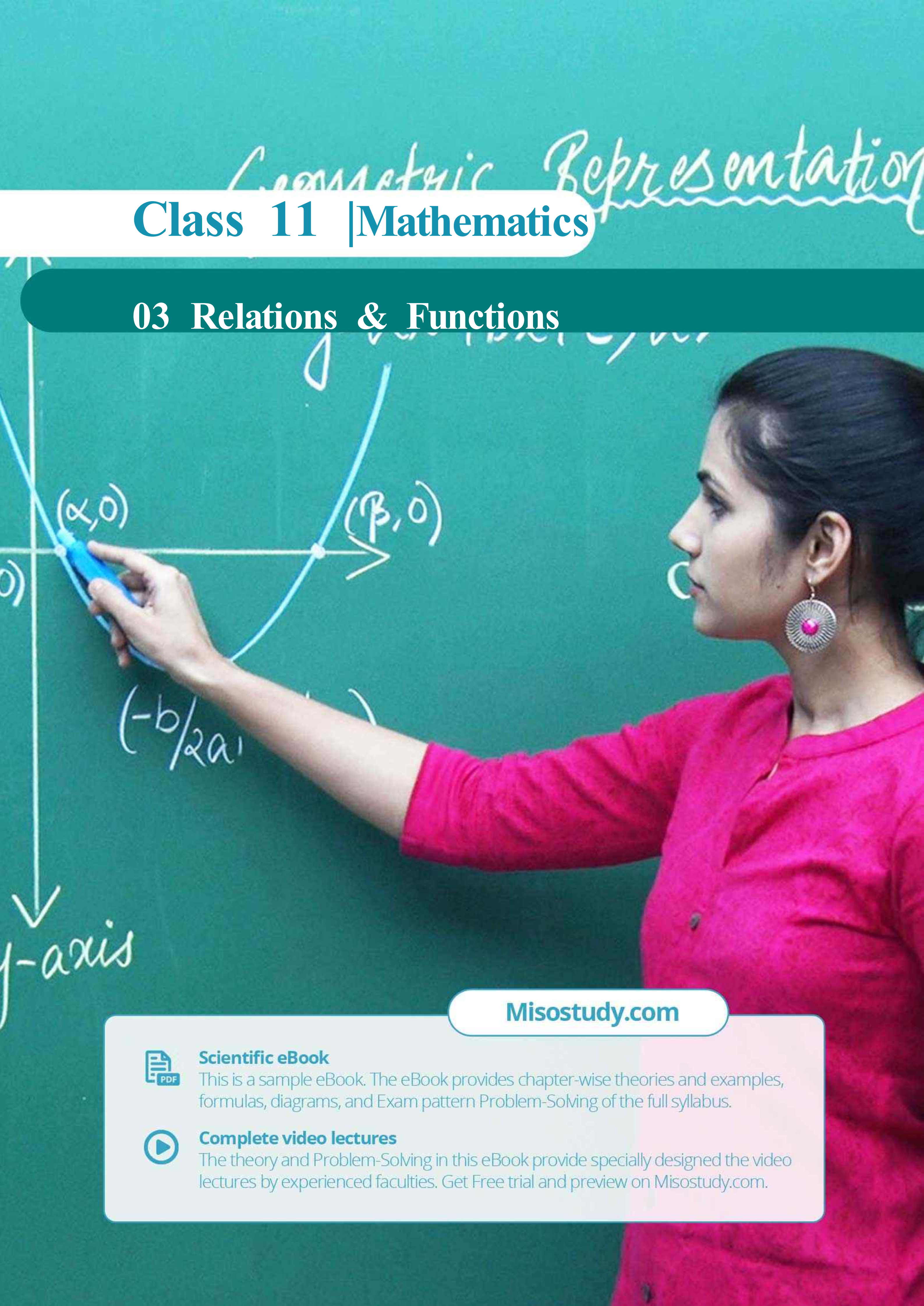

Class 11 |Mathematics

03 Relations & Functions

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01. Definition

(i) **ORDERED PAIR**

An ordered pair consists of two objects or elements in a given fixed order.
For example, if A and B are any two sets, then by an ordered pair of elements we mean pair (a, b) in that order, where $a \in A$, $b \in B$.

NOTE ☞ An ordered pair is not a set consisting of two elements. The ordering of the two elements in an ordered pair is important and the two elements need not be distinct.

(ii) **EQUALITY OF ORDERED PAIRS**

Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal iff

$$a_1 = a_2 \text{ and } b_2 = b_2$$

i.e., $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2$ and $b_2 = b_2$

Example *Find the values of a and b, if* $(3a - 2, b + 3) = (2a - 1, 3)$.

Solution By the definition of equality of ordered pairs, we have

$$(3a - 2, b + 3) = (2a - 1, 3)$$
$$\Leftrightarrow \quad 3a - 2 = 2a - 1 \text{ and } b + 3 = 3$$
$$\Leftrightarrow \quad a = 1 \text{ and } b = 0$$

(iii) **CARTESIAN PRODUCT OF SETS**

Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$

Example *If* $A = \{2, 4, 6\}$ *and* $B = \{1, 2\}$, *then*

$$A \times B = \{2, 4, 6\} \times \{1, 2\} = \{(2, 1), (2, 2), (4, 1), (4, 2), (6, 1), (6, 2)\}$$

and, $B \times A = \{1, 2\} \times \{2, 4, 6\} = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6)\}$

02. Number of Elements in the Cartesian Product of Two Sets

RESULT

If A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$.

PROOF

Let $A=\{a_1, a_2, a_3, \ldots, a_m\}$ and $B=\{b_1, b_2, b_3, \ldots, b_n\}$ be two sets having m and n elements respectively. Then,

$$\begin{aligned} A\times B=\{&(a_1,b_1),(a_1,b_2),(a_1,b_3), \ldots,(a_1,b_n)\\ &(a_2,b_1),(a_2,b_2),(a_2,b_3) \ldots,(a_2,b_n)\\ &\quad\vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad \vdots\\ &(a_m,b_1),(a_m,b_2),(a_m,b_3) \ldots,(a_m,b_n)\}\end{aligned}$$

Clearly, in the tabular representation of $A\times B$ there are m rows of ordered pairs and each row has n distinct ordered pairs.

So, $A\times B$ has mn elements.

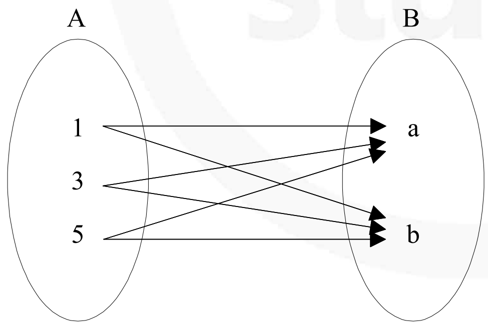
Hence, $n(A\times B)=mn=n(A)\times n(B)$

REMARK (i) If either A or B is an infinite set, then $A\times B$ is an infinite set.
(ii) If A, B, C are finite sets, then $n(A\times B\times C)=n(A)\times n(B)\times n(C)$

03. Diagramatic Representation of Cartesian Product of Two Sets

In order to represent $A\times B$ by an arrow diagram, we first draw Venn diagrams representing sets A and B one opposite to the other as shown in Figure. Now, we draw line segments starting from each element of A and terminating to each element of set B.

If $A=\{1,2,3\}$ and $B=\{a,b\}$, then following figure gives the arrow diagram of $A\times B$.



04. Some Useful Results

RESULT 1

For any three sets $A, B, C,$ prove that:

(i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

Proof (i) Let (a, b) be an arbitrary element of $A \times (B \cup C)$. Then,

$(a, b) \in A \times (B \cup C) \Rightarrow a \in A$ and $b \in B \cup C$ [by def.]

$\Rightarrow$ $a \in A$ and $(b \in B$ or $b \in C)$ [by def. of union]

$\Rightarrow$ $(a \in A$ and $b \in B)$ or $(a \in A$ and $b \in C)$

$\Rightarrow$ $(a, b) \in A \times B$ or $(a, b) \in A \times C \Rightarrow (a, b) \in (A \times B) \cup (A \times C)$

$\therefore$ $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cup (A \times C)$. Then,

$(x, y) \in (A \times B) \cup (A \times C) \Rightarrow (x, y) \in A \times B$ or $(x, y) \in A \times C$

$\Rightarrow$ $(x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow$ $x \in A$ $(y \in B$ or $y \in C)$

$\Rightarrow$ $x \in A$ and $y \in (B \cup C) \Rightarrow (x, y) \in A \times (B \cup C)$

$\therefore$ $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ...(ii)

Hence, from (i) and (ii), we have $A \times (B \cap C)$. Then,

Proof (ii) Let (a, b) be an arbitrary element of $A \times (B \cap C)$. Then,

$(a, b) \in A \times (B \cap C) \Rightarrow a \in A$ and $b \in (B \cap C)$ [by def.]

$\Rightarrow$ $a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow$ $(a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow$ $(a, b) \in A \times B$ and $(a, b) \in A \times C$ [by def.]

$\Rightarrow$ $(a, b) \in (A \times B) \cap (A \times C)$

$\therefore$ $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cap (A \times C)$. Then,

$(x, y) \in (A \times B) \cap (A \times C) \Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in A \times C$

$\Rightarrow$ $(x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow$ $x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow$ $x \in A$ and $y \in (B \cap C) \Rightarrow (x, y) \in A \times (B \cap C)$

$\therefore$ $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ...(ii)

Hence, from (i) and (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

RESULT 2

For any three sets $A, B, C,$ prove that:

$A\times(B-C)=(A\times B)-(A\times C)$

Proof Let (a, b) be an arbitrary element of $A \times (B - C)$. Then,

$(a, b) \in A \times (B - C) \Rightarrow a \in A$ and $b \in (B - C) \Rightarrow a \in A$ and $(b \in B$ and $b \notin C)$

$\Rightarrow$ $(a \in A$ and $b \in B)$ and $(a \in A$ and $b \notin C)$

$\Rightarrow$ $(a, b) \in (A \times B)$ and $(a, b) \notin (A \times C) \Rightarrow (a, b) \in (A \times B) - (A \times C)$

$\therefore$ $A \times (B - C) \subseteq (A \times B) - (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) - (A \times C)$. Then,

$(x, y) \in (A \times B) - (A \times C) \Rightarrow (x, y) \in A \times B$ and $(x, y) \notin A \times C$

$\Rightarrow$ $(x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C) \Rightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Rightarrow$ $x \in A$ and $y \in (B - C) \Rightarrow (x, y) \in A \times (B - C)$

$\therefore$ $(A \times B) - (A \times C) \subseteq A \times (B - C)$ ...(ii)

Hence, from (i) and (ii), we get

$A \times (B - C) = (A \times B) - (A \times C)$

RESULT 7

For any sets A, B, C, D prove that:

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Proof Let (a, b) be an arbitrary element of $(A \times B) \cap (C \times D)$. Then,

$(a, b) \in (A \times B) \cap (C \times D) \Rightarrow (a, b) \in A \times B$ and $(a, b) \in C \times D$

$\Rightarrow$ $(a \in A$ and $b \in B)$ and $(a \in C$ and $b \in D)$

$\Rightarrow$ $(a \in A$ and $b \in C)$ and $(a \in B$ and $b \in D)$

$\Rightarrow$ $a \in (A \cap C)$ and $b \in B \cap D \Rightarrow (a, b) \in (A \cap C) \times (B \cap D)$

$\therefore$ $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$

Similarly, $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$

Hence, $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Corollary For any sets A and B, prove that $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$.

06. Concept of Relations

(i) **RELATION**

Let A and B be tow sets. Then a relation R from A to B is a subset of $A \times B$. Thus, R is a relation form A to $B \Leftrightarrow R \subseteq A \times B$.

If R is a relation from a non-void set A to a non-void set B and if $(a, b) \in R$, then we write aRb which is read as $'a$ is related to b by the relation R'. If $(a, b) \notin R$, then we write aRb and we say that a is not related to b by the relation R.

(ii) **TOTAL NUMBER OF RELATIONS**

Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subsets of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines a relation from A to B, so total numbers of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

07. Representation of a Relation

A relation from a set A to a set B can be represented in any one of the following forms:

(i) **ROSTER FORM**

In this form a relation is represented by the set of all ordered pairs belonging to R.

Example If R is a relation from set $A = \{-2, -1, 0, 1, 2\}$ to set $B = \{0, 1, 4, 9, 10\}$ by the rule

$$a\ R\ b \Leftrightarrow a^2 = b$$

Then, $0\ R\ 0$, $-2\ R\ 4$, $-1\ R\ 1$, $1\ R\ 1$ and $2\ R\ 4$.

So, R can be described in Roster form as follows:

$$R = \{(0, 0), (-1, 1), (-2, 4), (1, 1), (2, 4)\}$$

(ii) **SET-BUILDER FORM**

In this form the relation R from set A to set B is represented as

$R = \{(a, b) : a \in A, b \in B$ and a, b satisfy the rule which associates a and $b\}$

Example If $A = \{1, 2, 3, 4, 5\}$, $B = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\right\}$ and R is a relation from A to B given by

$$R = \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$$

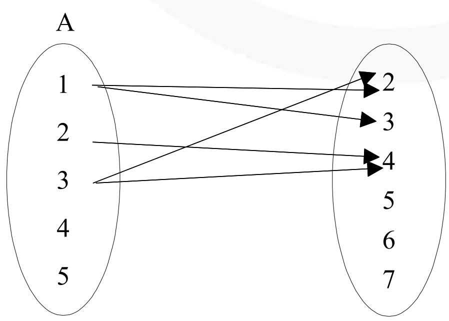
Then, R in set-builder form can be described as follows:

$$R = \left\{(a, b) : a \in A, b \in B \text{ and } b = \frac{1}{a}\right\}$$

(iii) **BY ARROW DIAGRAM**

In order to represent a relation from set A to a set B by an arrow diagram, we draw arrows from first components to the second components of all ordered pairs belonging to R.

Example Relation R = {(1, 2), ((2, 4), (3, 2), (1, 3), (3, 4)} from set $A = \{1, 2, 3, 4, 5\}$ to set $B = \{2, 3, 4, 5, 6, 7\}$ can be represented by the following arrow diagram:



08. Domain and Range of a Relation

Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain or R, while the set of all second components or coordinates of the ordered pairs in R is called the range or R.

Thus, **Dom** $(R) = \{a : (a, b) \in R\}$ **and Range** $(R) = \{b : (a, b) \in R\}$.

★ **RELATION ON A SET** Let A be a non-void set. Then, a relation from A to itself i.e. a subset of $A \times A$, is called a relation on set A.

Example If $A = \{1, 3, 5, 7\}$, $B = \{2, 4, 6, 8, 10\}$ and let $R = \{(1, 8), (3, 6), (5, 2), (1, 4)\}$ be a relation from A to B. Then,

Dom $(R) = \{1, 3, 5\}$ and Range $(R) = \{8, 6, 2, 4\}$

10. Function as a Special Kind of Relation

DEFINITION

Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B, if

(i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$

(ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.

11. Function as a Correspondence

DEFINITION

Let A and B be two non-empty sets. Then a function $'f'$ from set A to st B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated to elements in set B.

(ii) an element of set A is associated to a unique element in set B.

In other words, a function $'f'$ from a set A to a set B associates each element of set A to a unique element of set B.

NOTE ☞ If and element $a \in A$ is associated to an element $b \in B$, then b is called 'the f-image of a' or 'image of a under f' or 'the value of the function f at a'. Also, a is called the pre-image of b under the function f. We write it as : $b = f(a)$

12. Description of a Function

Let $f: A \to B$ be a function such that the set A consists of a finite number of elements. Then, $f(x)$ be described by listing the values which it attains at different points of its domain. For example, if $A = \{-1, 1, 2, 3\}$ and B is the set of real numbers, then a function $f: A \to B$ can be described as $f(-1) = 3, f(1) = 0, f(2) = 3/2$ and $f(3) = 0$. In case, A is an infinite set, then f cannot be described by listing the image at points in its domain. In such cases functions are generally described by a formula. For example, $f: Z \to Z$ given by $f(x) = x^2 + 1$ or $f: R \to R$ given by $f(x) = e^x$ etc.

13. Domain, Co-Domain and Range of a Function

Let $f: A \to B$. Then, the set A is known as the domain of f and the set B is known as the co-domain of f. The set of all f-images of elements of A is known as the range of f or image set of A under f and is denoted by $f(A)$.
Thus, $f(A) = \{f(x) : x \in A\} =$ Range of f
Clearly, $f(A) \subseteq B$.

14. Equal Function

DEFINITION
Two functions f and g are said to be equal iff
(i) domain of $f =$ domain of g,
(ii) co-domain of $f =$ co-domain of g,
and (iii) $f(x) = g(x)$ for every x belonging to their common domain.
If two function f and g are equal, then we write $f = g$.

Example *Let $A = \{1, 2\}$, $B = \{3, 6\}$ and $f: A \to B$ given by $f(x) = x^2 + 2$ and $g: A \to B$ given by $g(x) = 3x$. Then, we observe that f and g have the same domain and co-domain. also we have, $f(1) = 3 = g(1)$* and *$f(2) = 6 = g(2)$*
Hence, $f = g$.

15. Real Valued Function

A function $f: A \to B$ is called a real valued function, if B is a subset of R (set of all real numbers).
If A and B both are subsets of R, then f is called a real function.

NOTE ☞ In practice, real functions are described by giving the general expression or formula describing it without mentioning its domain and co-domain.

16. Domain of Real Functions

The domain of $f(x)$ is the set of all those real numbers for which $f(x)$ is meaningful.

17. Range of Real Functions

The range of a real function of a real variable is the set of all real values taken by $f(x)$ at points in its domain. In order to find the range of a real function $f(x)$, we may use the following algorithm.

ALGORITHM

STEP I — Put $y = f(x)$

STEP II — Solve the equation $y = f(x)$ for x in terms of y. Let $x = \phi(y)$.

STEP III — Find the values of y for which the values of x, obtained from $x = \phi(y)$, are real and in the domain of f.

STEP IV — The set of values of y obtained in step *III* is the range of f.

Example *Find the domain and range of the function f (x) given by*

$$f(x) = \frac{x-2}{3-x}$$

Solution We have,

$$f(x) = \frac{x-2}{3-x}$$

Domain of f : Clearly, $f(x)$ is defined for all x satisfying $3 - x \neq 0$ i.e. $x \neq 3$.
Hence, Domain $(f) = R - \{3\}$.
Range of f : Let $y = f(x)$, i.e.

$$y = \frac{x-2}{3-x}$$

$\Rightarrow$ $3y - xy = x - 2$

$\Rightarrow$ $x(y + 1) = 3y + 2$

$\Rightarrow$ $x = \dfrac{3y+2}{y+1}$

Clearly, x assumes real values for all y except $y + 1 = 0$ i.e. $y = -1$.
Hence, Range $(f) = R - \{-1\}$.

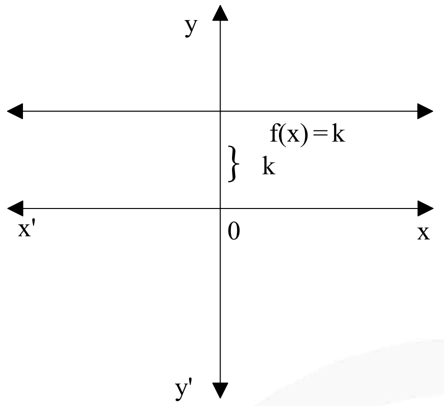
18. Some Standard Real Functions and Their Graphs

CONSTANT FUNCTION

If k is a fixed real number, then a function $f(x)$ given by

$$f(x)=k \text{ for all } x \in R$$

is called a constant function.



$y=k.$

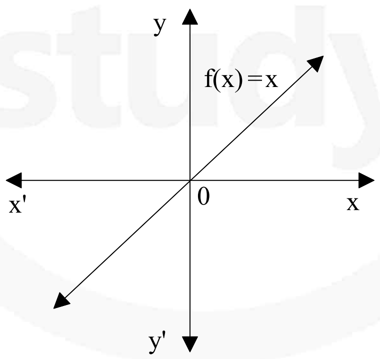
IDENTITY FUNCTION

The function that associates each real number to itself is called the identity function and is usually denoted by I.

Thus, the function $I: R \rightarrow R$ defined by

$$I(x)=x \text{ for all } x \in R$$

is called the identity function.



$y=x.$

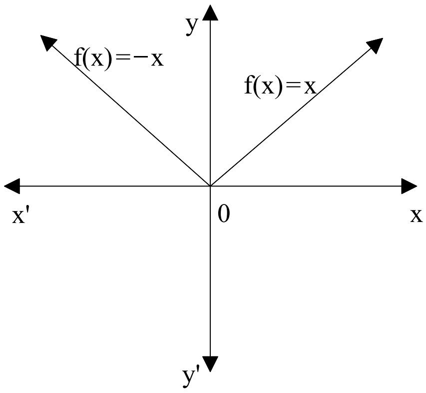
MODULUS FUNCTION

The function $f(x)$ defined by

$$f(x)=|x|=\begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$$

is called the modulus function.

It is also called the absolute value function.



$y = |x|$.

GREATEST INTEGER FUNCTION (FLOOR FUNCTION)

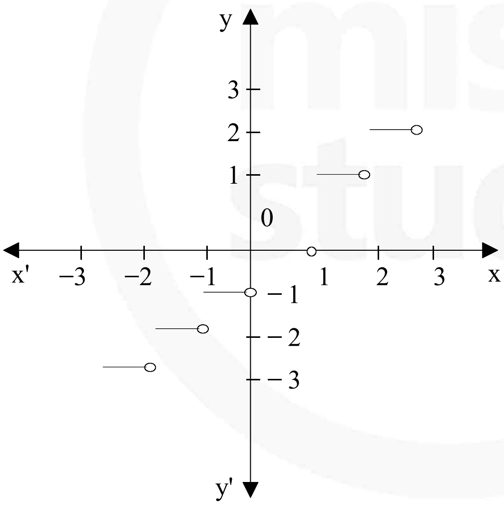
For any real number x, we use the symbol $[x]$ or, $\lfloor x \rfloor$ to denote the greatest integer less than or equal to x.

The function $f : R \to R$ defined by

$$f(x) = [x] \text{ for all } x \in R$$

is called the greatest integer function or the floor function.

It is also called a step function.



$y = [x]$.

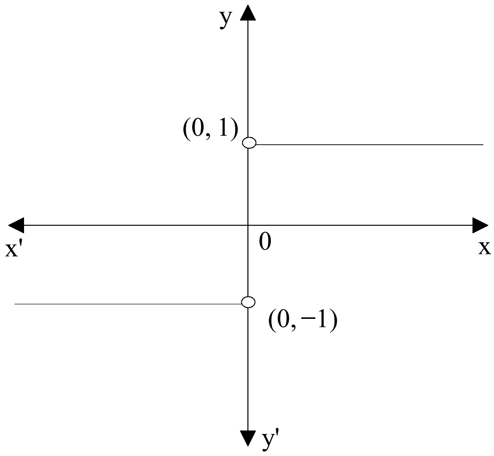
SIGNUM FUNCTION

The function f defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

or, $$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

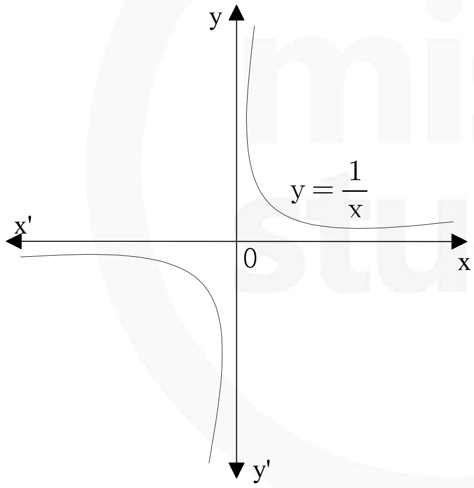
is called the signum function.



$y = \frac{|x|}{x}$.

RECIPROCAL FUNCTION

The function $f : R - \{0\} \rightarrow R$ defined by $f(x) = \frac{1}{x}$ is called the reciprocal function.



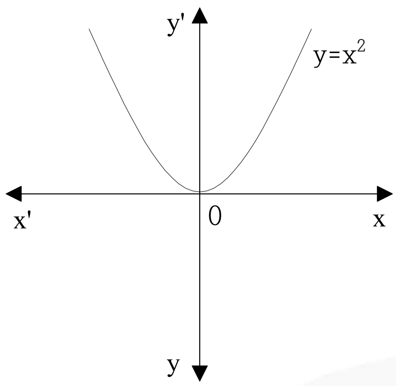
$y = \frac{1}{x}$

SQUARE FUNCTION

The function $f: R \to R$ defined by

$$f(x) = x^2$$

is called the square function.



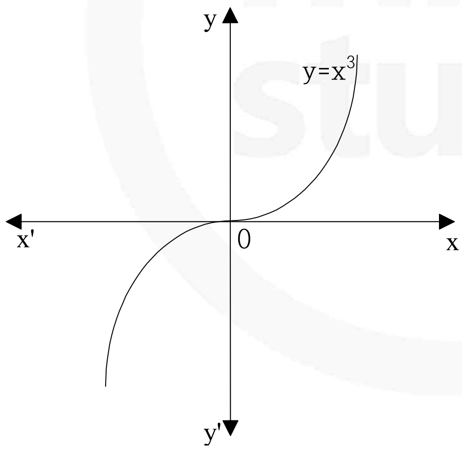
$y = x^2$

CUBE FUNCTION

The function $f: R \to R$ defined by

$$f(x) = x^3$$

is called the cube function.



$y = x^3$

REMARK (1) A function $f: R \to R$ is said to be a polynomial function if $f(x)$ is a polynomial in x. For example, $f(x) = x^2 - x + 4$, $g(x) = x^3 + 3x^2 + \sqrt{2}x - 1$ etc are polynomial functions.

(2) A function of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$, is called a rational function. The domain of a rational function $f(x) = \frac{p(x)}{q(x)}$ is the set of all real numbers, except points where $q(x) = 0$.

19. REAL FUNCTIONS

OPERATIONS ON REAL FUNCTIONS-

In this section, we shall introduce various operations namely addition, subtraction, multiplication, division etc. on real function.

ADDITION

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then, their sum $f+g$ is defined as that function from $D_1 \cap D_2$ to R which associates each $x \in D_1 \cap D_2$ to be number $f(x)+g(x)$.

PRODUCT

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then, their product (or pointwise multiplication) fg is a function from $D_1 \cap D_2$ to R and is defined as

$$(fg)(x) = f(x)g(x) \text{ for all } x \in D_1 \cap D_2$$

DIFFERENCE (SUBTRACTION)

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then the difference of g from f is denoted by $f-g$ and is defined as

$$(f-g)(x) = f(x) - g(x) \text{ for all } x \in D_1 \cap D_2$$

QUOTIENT

Let$f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then the quotient of f by g is denoted by $\frac{f}{g}$ and it is a function from $D_1 \cap D_2 - \{x : g(x) = 0\}$ to R defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for all } x \in D_1 \cap D_2 - \{x : g(x) = 0\}$$

MULTIPLICATION OF A FUNCTION BY A SCALAR

Let $f : D \to R$ be a real function and α be a scalar (real number). Then the product αf is a function from D to R and is defined as

$$(\alpha f)(x) = \alpha f(x) \text{ for all } x \in D.$$

RECIPROCAL OF A FUNCTION

If $f : D \to R$ is a real function, then its reciprocal function $\frac{1}{f}$ is a function from $D - \{x : f(x) = 0\}$ to R and is defined as

Example I $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

Example I Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions defined over the set of non-negative real numbers. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

Solution We have,

$$(f + g)(x) = x^2 + 2x + 1,\ (f - g)(x) = x^2 - 2x - 1,$$

$$(fg)(x) = x^2(2x + 1) = 2x^3 + x^2,\ \left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1},\ x \neq -\frac{1}{2}$$

REMARK (1) The sum, difference product and quotient are defined for real functions only on their common domain. These operations do not make any sense for general functions even if their domains are same, because the sum, difference, product and quotient may or may not be meaningful for the elements in their common domain.

(2) For any real function $f: D \rightarrow R$ and $n \in N$, we define

$$\underbrace{(fff \ldots f)}_{n\text{-times}}(x) = \underbrace{f(x)f(x) \ldots f(x)}_{n\text{-times}} = \{f(x)\}^n \text{ for all } x \in D$$

CBSE Exam Pattern Exercise

Subjective Questions (1)

(Q 1 to 2) One Mark

1. Let $f : R \rightarrow R$ be given by $f(x) = x^2 + 3$. Find the pre-images of 39 and 2 under f.

2. If $f(x) = 3x^3 - 5x^2 + 9$, find $f(x - 1)$.

(Q 3 to 4) Two Mark

3. Let $A = \{1, 2, 3\}$ and $B = \{x : x \in N, x$ is prime less than $5\}$. Find $A \times B$ and $B \times A$ & $((A \times B) \cap (B \times A))$

4. *Is* $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ *a function? If this is described by the formula,* $g(x) = \alpha x + \beta$, *then what values should be assigned to* α *and* β?

(Q 5 to 7) Four Marks

5. *Let* $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on set A by $R = \{(x, y) : y = x + 1\}$
   (i) Depict this relation using an arrow diagram.
   (ii) Write down the domain, co-domain and range of R.

6. Find the domain and range of each of the following function: $f(x) = \dfrac{3}{2-x^2}$.

7. *Let* A *be a non-empty set such that* $A \times B = A \times C$. *Show that* $B = C$.

(Q 8 to 10) Four Marks

8. *Let* f *and* g *be real functions defined by* $f(x) = \sqrt{x+2}$ *and* $g(x) = \sqrt{4-x^2}$. *Then, find each of the following functions*:
   (i) $f + g$ (ii) $f - g$ (iii) fg
   (iv) $\dfrac{f}{g}$ (v) ff (vi) gg

9. The function f is defined by $f(x) = \begin{cases} 1-x, & x<0 \\ 1 , & x=0 \\ x+1, & x>0 \end{cases}$ Draw the graph of $f(x)$.

10. Define the function $f : \mathbf{R} \rightarrow \mathbf{R}$ by $y = f(x) = x^2$, $x \in \mathbf{R}$. Complete the Table given below by using this definition. What is the domain and range of this function? Draw the graph of f.

x	–4	–3	–2	–1	0	1	2	3	4
$y = f(x) = x^2$									

Answer & Solution

Q1.
Let x be the pre-image of 39. Then,

$f(x) = 39 \Rightarrow x^2 + 3 = 39 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

So, pre-images of 39 are – 6 and 6.

Let x be the pre-image of 2. Then,

$f(x) = 2 \Rightarrow x^2 + 3 = 2 \Rightarrow x^2 = -6$

Q2.
$3x^3 - 5x^2 + 9$
$3(x - 1)^3 - 5(x - 1)^2 + 9$
$3(x^3 - 1 - 3x(x - 1)) - 5(x^2 + 1 - 2x) + 9$
$3(x^3 - 1 - 3x^2 + 3x) - 5x^2 - 5 + 10x + 9$
$3x^3 - 3 - 9x^2 + 9x - 5x^2 - 5 + 10x + 9$
$3x^3 - 14x^2 + 19x + 1$

Q3.
We have,

$A = \{1, 2, 3\}$

and, $B = \{x : x \in N, x \text{ is prime less than } 5\} = \{2, 3\}$

$\therefore$ $A \times B = \{1, 2, 3\} \times \{2, 3\} = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$

and, $B \times A = \{2, 3\} \times \{1, 2, 3\} = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

$(A \times B) \cap (B \times A)$

$= \{(2, 2), (2, 3,), (3, 2), (3, 3)\}$

Q4.
Since no two ordered pairs in g have the same first component. So, g is a function such that $g(1) = 1$, $g(2) = 3$, $g(3) = 5$ and $g(4) = 7$.

It is given that $g(x) = \alpha(x) = \alpha x + \beta$.

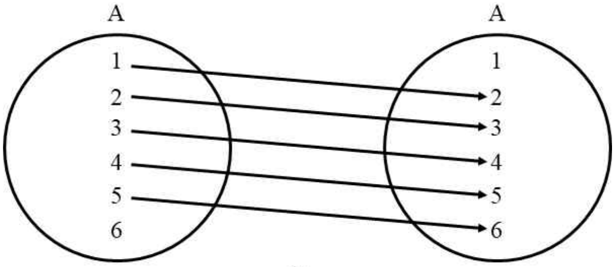
$\therefore$ $g(1) = 1$ and $g(2) = 3 \Rightarrow \alpha + \beta = 1$ and $2\alpha + \beta = 3 \Rightarrow \alpha = 2, \beta = -1$.

Q5.
(i) Putting $x = 1, 2, 3, 4, 5, 6$ in $y = x + 1$, we get $y = 2, 3, 4, 5, 6, 7$.

For $x = 6$, we get $y = 7$ which does not belong to set A.

$\therefore$ R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

The arrow diagram representing R is as follows:



Figure

(ii) Clearly, Domain (R) = {1, 2, 3, 4, 5}, Range (R) = {2, 3, 4, 5, 6}.

Q6.

We have,

$$f(x) = \frac{3}{2 - x^2}$$

For $f(x)$ to be real, we must have

$2 - x^2 \neq 0 \Rightarrow x \Rightarrow \pm \sqrt{2}$.

$\therefore$ Domain $(f) = R - \{-\sqrt{2}, \sqrt{2}\}$

Let $y(x) = y$. Then,

$y = f(x)$

$\Rightarrow y = \dfrac{3}{2 - x^2}$

$\Rightarrow 2y - x^2y = 3$

$\Rightarrow x^2y = 2y - 3$

$\Rightarrow x = \pm \dfrac{\sqrt{2y - 3}}{y}$

Now, x will take real values other than $-\sqrt{2}$ and $\sqrt{2}$, if

$\dfrac{2y - 3}{y} \geq 0$

Figure

$\Rightarrow y \in (-\infty, 0) \cup \left[\dfrac{3}{2}, \infty\right)$

Hence, range $(f) = (-\infty, 0) \cup [3/2\ \infty)$.

Q7.

Let b be an arbitrary element of B. Then,

$(a, b) \in A \times B$ for all $a \in A$

$\Rightarrow (a, b) \in A \times C$ for all $a \in A$ $[\because A \times B = A \times C]$

$\Rightarrow b \in C$

Thus, $b \in B \Rightarrow b \in C$

$\therefore B \subset C$ ...(i)

Now, let c be an arbitrary element of C. Then,

$(a, c) \in A \times C$ for all $a \in A$ $\quad [\because A \times B = A \times C]$

$\Rightarrow (a, c) \in A \times B$ for all $a \in A$

$\Rightarrow c \in B$

Thus, $c \in C \Rightarrow c \in B$

$\therefore C \subset B$ ...(ii)

From (i) and (ii), we get

$B = C.$

Q8.

We have,

$$f(x) = \sqrt{x+2} \text{ and } g(x) = \sqrt{4-x^2}$$

Clearly, $f(x)$ is defined for

$x + 2 \geq 0 \Rightarrow x \geq -2 \Rightarrow x \in [-2, \infty)$

$\therefore$ Domain $(f) = [-2, \infty]$

$g(x)$ is defined for

$4 - x^2 \geq 0 \Rightarrow x^2 - 4 \leq 0 \Rightarrow (x - 2)(x + 2) \leq 0 \Rightarrow x \in [-2, 2]$

$\therefore$ Domain $(g) = [-2, 2]$

Now,

Domain $(f) \cap$ Domain $(g) = [-2, \infty) \cap [-2, 2] = [-2, 2]$

(i) $f + g : [-2, 2] \to R$ is given by

$(f + g)(x) = f(x) + g(x) = \sqrt{x+2} + \sqrt{4-x^2}$

(ii) $f - g : [-2, 2] \to R$ is given by

$(f - g)(x) = f(x) - g(x) = \sqrt{x+2} - \sqrt{4-x^2}$

(iii) $fg : [-2, 2] \to R$ is given by

$(fg)(x) = f(x)\, g(x)$

$\Rightarrow (fg)(x) = \sqrt{x+2} - \sqrt{4-x^2}$

$\Rightarrow (fg)(x) = \sqrt{(x+2)^2(2-x)} = (x+2)\sqrt{2-x}$

(iv) We have,

$g(x) = \sqrt{4-x^2}$

$\therefore g(x) = 0 \Rightarrow 4 - x^2 = 0 \Rightarrow x = \pm 2.$

So, Domain $\left(\frac{f}{g}\right) = [-2, 2] - [-2, 2] = (-2, 2)$

$\therefore \frac{f}{g} : (-2, 2) \to R$ is given by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{(x+2)}}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2-x}}$$

(v) We have,

$(ff)(x) = f(x)\, f(x) = [f(x)]^2 = \left(\sqrt{x+2}\right)^2 = x + 2$ for all $x \in [-2, \infty]$

(vi) We have,

$(gg)(x) = g(x)\, g(x) = [g(x)]^2 = \left(\sqrt{4-x^2}\right)^2 = 4 - x^2$ for all $x \in [-2, 2]$

Q9.

Here, $f(x) = 1 - x,\ x < 0$, this gives

$f(-4) = 1 - (-4) = 5$;

$f(-3) = 1 - (-3) = 4$,

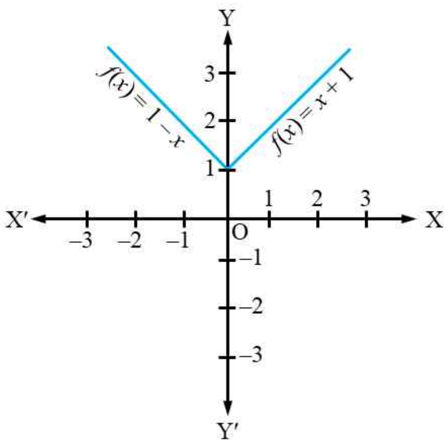
$f(-2) = 1 - (-2) = 3$

$f(-1) = 1 - (-1) = 2$; etc,

and $f(1) = 2,\ f(2) = 3,\ f(3) = 4$

$f(4) = 5$ and so on for $f(x) = x + 1,\ x > 0$.

Thus, the graph of f is as shown in Figure.



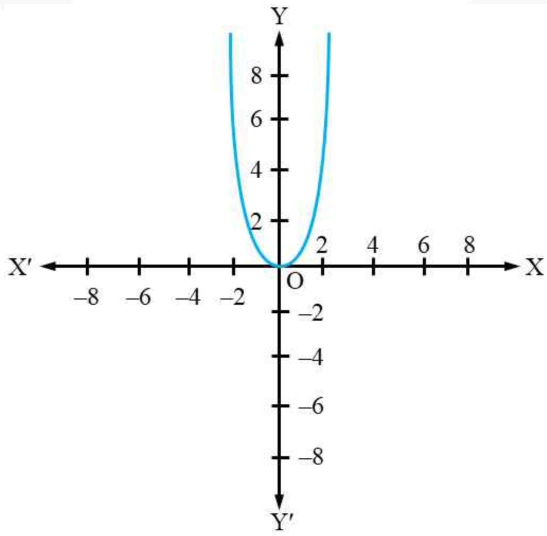
Figure

Q10.

The completed Table is given below:

x	–4	–3	–2	–1	0	1	2	3	4
$y = f(x) = x^2$	16	9	4	1	0	1	4	9	16

Domain of $f = \{x : x \in \boldsymbol{R}\}$. Range of $f = \{x2 : x \in \boldsymbol{R}\}$. The graph of f is given by Figure.



Figure