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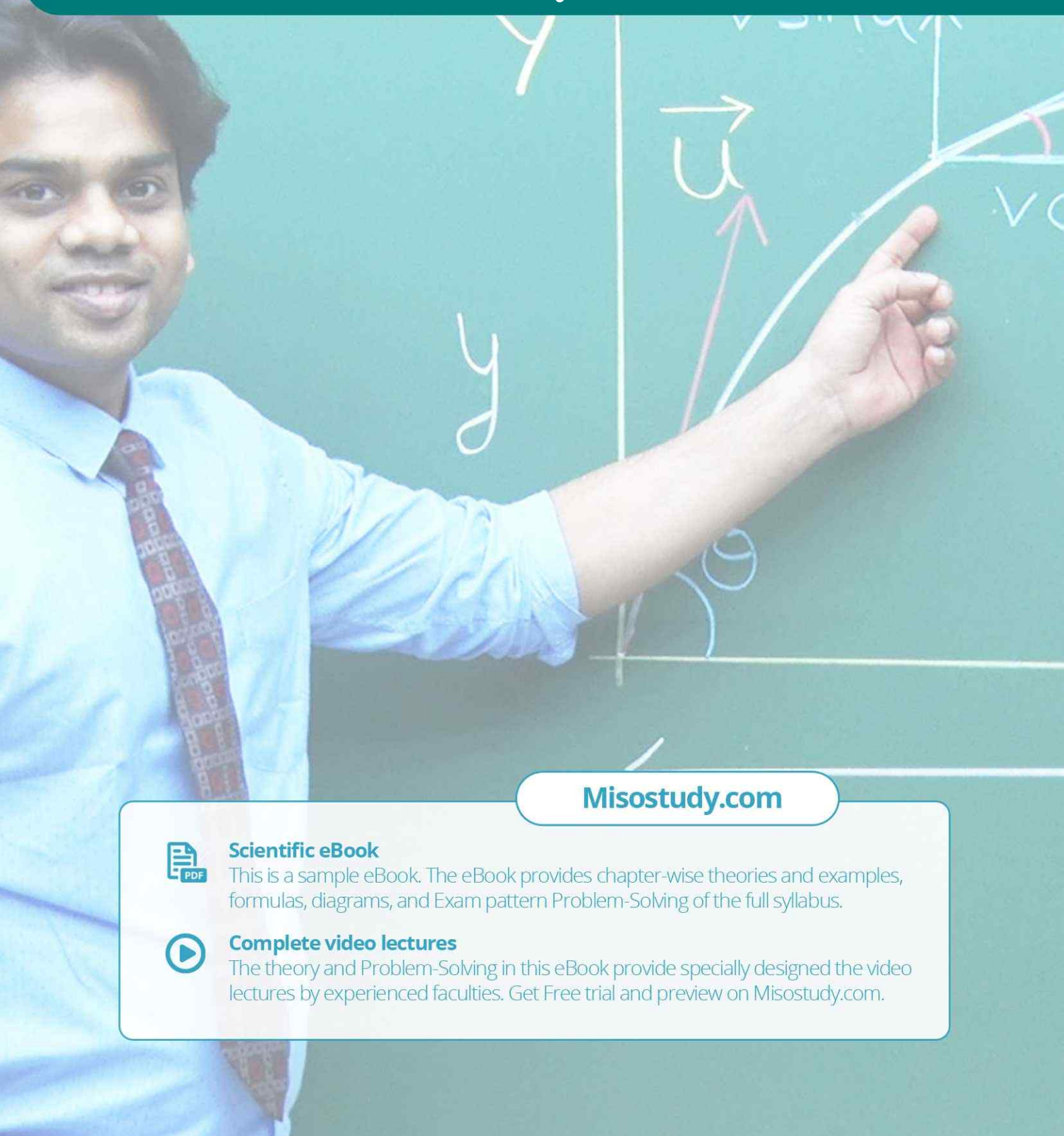
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Class 12 | Physics

02 Communication System



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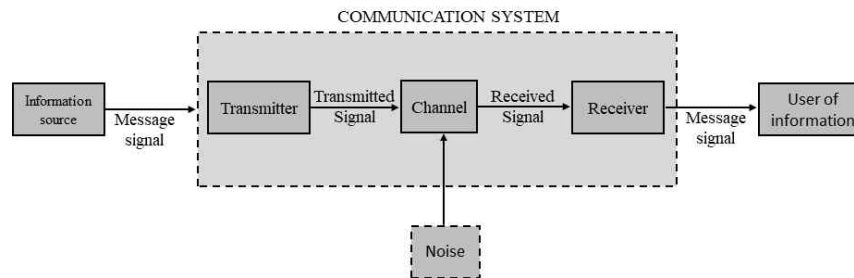
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01. Introduction

The act of transmission and reception of information is known as communication.

02. Elements of a Communication System

Every communication system has three essential elements, transmitter, medium and receiver.



There are two basic modes of communication: point-to-point and broadcast.

03 Basic Terminology of Communication Systems

- (i) **Transducer** : Device that converts one form of energy into another.
- (ii) **Signal**: Information converted in electrical form and suitable for transmission. Signals can be either *analog or digital*.
- (iii) **Noise**: The unwanted signals that tend to disturb the transmission and processing of message signals
- (iv) **Transmitter**: Processes the incoming message signal so as to make it suitable for transmission through a channel and subsequent reception
- (v) **Receiver**: Extracts the desired message signals from the received signals at the channel output.
- (vi) **Attenuation**: It is the loss of strength a signal while propagating through a medium.
- (vii) **Amplification**: The process of increasing the amplitude of a signal using an electronic circuit called the amplifier.
- (viii) **Range** : Largest distance between a source and a destination up to which the signal is received with sufficient strength
- (ix) **Bandwidth**: Frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

- (x) **Modulation:** Original low frequency message/information signal cannot be transmitted to long distances because of obvious reasons. Therefore, at the transmitter, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information
- (xi) **Demodulation:** The process of extraction of information from the carrier wave at the receiver
- (xii) **Repeater:** A combination of receiver and a transmitter. Communication satellite is essentially a repeater station ins space.

NOTE ☞ Undesirable effects in the course of signal transmission are

- (i) **Attenuation** : decrease in signal strength due to energy loss.
- (ii) **Distortion** : waveform perturbation
- (iii) **Interference** : contamination by extraneous signals.
- (iv) **Noise** : due to random electrical signal

03. Types of Transmission Media

Broadly, transmission media have been divided into two types

- (i) **Guided transmission medium:** That communication medium of channel which is used in point to point communication between a single transmitter and receiver.
- (ii) **Unguided transmission medium:** communication medium which is used, where there is no point to point contact between the transmitter and receiver.

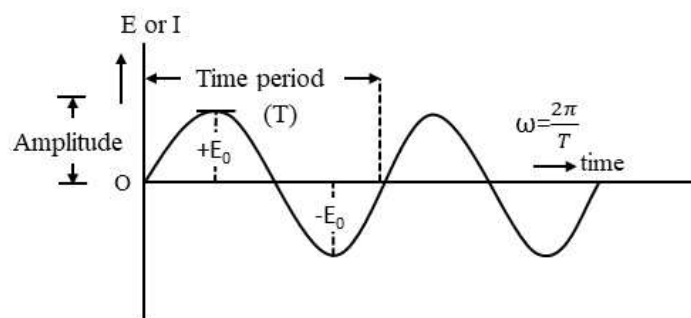
Characteristics and quality of transmission medium depends upon

- (i) Nature of transmission medium
- (ii) Nature of signal

The electrical signals are of two types:

- (i) **Analog signals:** An analog signal is that in which current or voltage value varies continuously with time

$$E = E_0 \sin(\omega t + \phi)$$



Examples of Analog signals are speech, music, sound produced by a vibrating tuning fork.

- (ii) **Digital signals:** A digital signal is discontinuous function of time, in contrast to an analog signal, wherein current or voltage value varies continuously with time.

Examples of Digital signals are (i) letters printed in a book (ii) listing of any data (iii) output of a digital computer (iv) electronic transmission of document at a distant place via telephone

04. Propagation of Electromagnetic Waves

An antenna at the transmitter in communication using radio waves, radiates the electromagnetic waves which travel through space and reach the receiving antenna at the other end.

05. Ground Wave

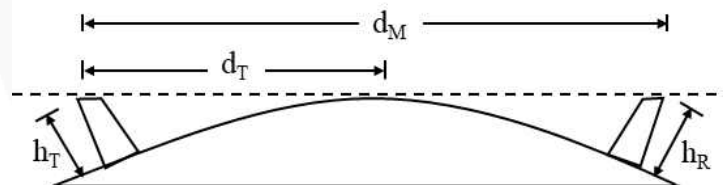
The antennas should have a size comparable to wavelength of signal. At longer wavelengths the antennas have large physical size and they are located on or very near to the ground.

06. Sky Waves

Long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection.

07. Space wave

A space wave travels in a straight line from transmitting antenna to the receiving antenna. Space waves are used for line-of-sight (LOS) communication as well as satellite communication.



Where R is the radius of the earth

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_R is the height of receiving antenna.

08. Determination of range

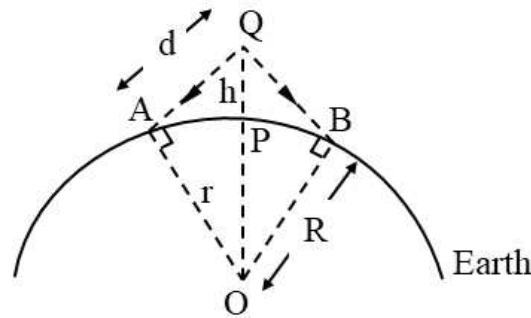
The range is determined by the height of transmitting antenna. The range AP or PB can be easily calculated by geometrical consideration. Suppose height of the tower is h and the radius of earth is r (that is $OA = OB = OP = r$). in the right-angled triangle OQA , we have

$$OQ^2 = QA^2 + OA^2$$

$$\therefore QA \approx AP = d$$

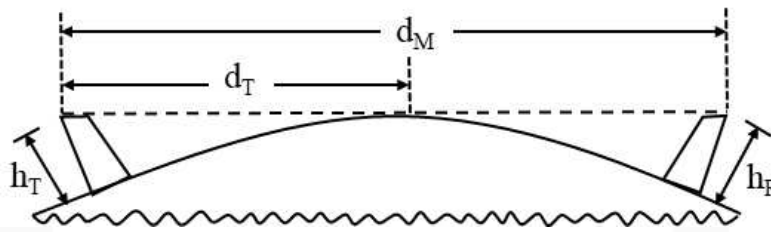
Therefore

$$\begin{aligned} (R+h)^2 &= R^2 + d^2 \\ R^2 + h^2 + 2Rh &= R^2 + d^2 \\ \therefore R^2 &\gg h^2 \\ \therefore d^2 &= 2Rh \\ d &= \sqrt{2Rh} \end{aligned}$$



For a height of 500 m, $d = 80$ km

Hence maximum line-of-sight distance between two antennas will be calculated as follows



$$d_M = \sqrt{2R \cdot h_T} + \sqrt{2R \cdot h_R}$$

Where d_M : Maximum line-of-sight distance between two antennas

d_T : Radio horizon of transmitting antenna

h_T : Height of transmitting antenna

h_R : Height of receiving antennas

09. Modulation And its Necessity

Size of the antenna or aerial : An antenna or aerial is needed, both for transmission and reception. Each antenna should have a size comparable to the wavelength of the signals, (at least $\lambda/4$ in size) so that time variation of the signal is properly sensed by the antenna.

Effective power radiated by antenna : Theoretical studies reveal that power P radiated from a linear.

antenna of length l is proportional $(l/\lambda)^2, i.e., P \propto \left(\frac{l}{\lambda}\right)^2$

Modulation is the phenomenon of superimposing the low audio frequency baseband message or information signals (called the modulation signals) on a high frequency wave (called, the carrier wave). The resultant wave is called the modulated wave, which is transmitted.

10. Amplitude Modulation

The amplitude modulation is produced by varying the amplitude of the carrier waves in accordance with the amplitude of the modulating wave (audio signal). Let the instantaneous values of the voltage of the carrier waves and the modulating signal be represented by

$$e_c = E_c \sin \omega_c t \quad \dots(i)$$

and $e_m = E_m \sin \omega_m t \quad \dots(ii)$

respectively. Here, $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$ are the angular frequencies of the carrier waves and the modulating signal respectively.

The instantaneous voltage of the modulated signal is given by

$$e = (E_c + E_m \sin \omega_m t) \sin \omega_c t$$

or $e = E_c \left(1 + \frac{E_m}{E_c} \sin \omega_m t \right) \sin \omega_c t \quad \dots(iii)$

In amplitude modulation, the degree of modulation is defined by a term called **modulation index**, which is given by

$$m_a = \frac{E_m}{E_c} \quad \dots(iv)$$

The modulation index is also called **modulation factor** or **depth of modulation**.

Therefore, the equation (iii) becomes

$$e = E_c (1 + m_a \sin \omega_m t) \sin \omega_c t$$

or $e = (E_c + m_a E_c \sin \omega_m t) \sin \omega_c t \quad \dots(v)$

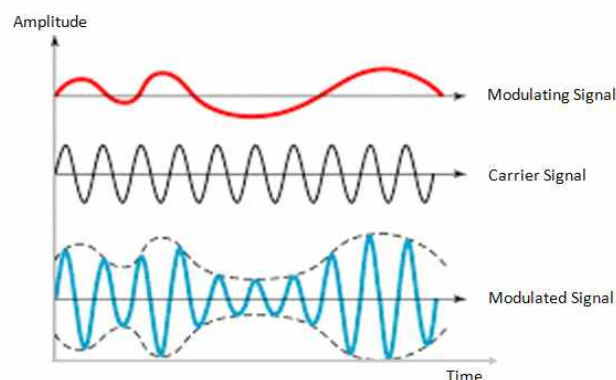
or $e = E \sin \omega_c t,$

where $E = E_c + m_a E_c \sin \omega_m t \quad \dots(vi)$

represents the amplitude of the modulated signal. It follows that the amplitude of the modulated signal varies with time in accordance with the amplitude of the modulating signal, Fig. gives the sketch of amplitude modulated signal with time. The amplitude of the modulated signal varies between $E_{\min} (= E_c - E_m)$ and $E_{\max} (= E_c + E_m)$. It can be easily proved that

$$m_a = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} \quad \dots(vii)$$

It follows that the expression for instantaneous value of the voltage of the modulated signal consists of three terms. The first term represents a wave form of carrier frequency ω_c and the second term of frequency $\omega_c - \omega_m$ (slightly less than ω_c), known as **lower sideband**. The third term represents a wave form of frequency $\omega_c + \omega_m$ (slightly greater than ω_c), called the **upper sideband**.



Bandwidth :

The modulated signal lies in the frequency range from $\omega_c - \omega_m$ to $\omega_c + \omega_m$ i.e. $2\omega_m$. It is called the bandwidth of the modulated signal.

Thus,

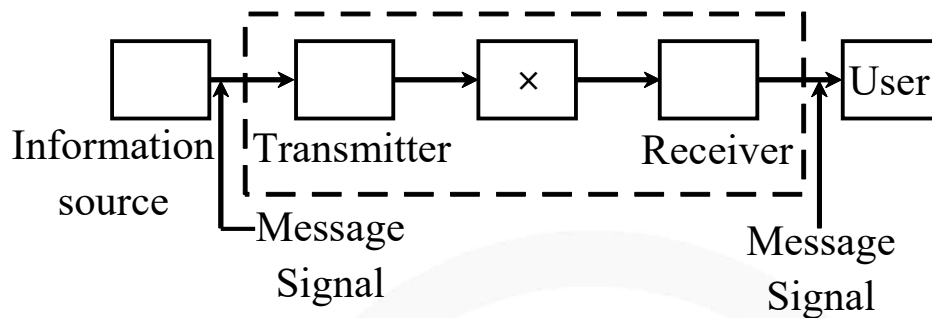
$$\text{band width} = 2 \times \text{frequency of modulating signal}$$



CBSE Exam Pattern Exercise Subjective Questions (1)

(Q 1 to 3) One Mark

1. The figure given below shows the block diagram of a generalised communication system. Identify the element labelled X and write its function.



2. Why are broadcast frequencies (carrier waves) sufficiently spaced in amplitude modulated wave?
3. How does the effective power radiated from a linear antenna depend on the wavelength of the signal to be transmitted?

(Q 4 to 7) Two Marks

4. Mention the function of any two of the following used in communication system.
- Transducer
 - Repeater
 - Transmitter
 - Bandpass filter
5. A message signal of frequency 10 kHz and peak voltage 10 V is used to modulate a carrier of frequency 1 MHz and peak voltage 20 V.
Determine
- the modulation index
 - the side bands produced.
- 6.
- What is line of sight communication?
 - Why is it not possible to use sky wave propagation of transmission of TV signals?
7. Define the term modulation. Draw a block diagram of a simple modulator for obtaining AM signal.

(Q 8 to 10) Three Marks

- 8.
- How is amplitude modulation achieved?
 - The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz, respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation?
9. Optical communication system having an operating wavelength λ (in meters) can use only $x\%$ of its source frequency as its channel bandwidth. The system is to be used for transmitting TV signals requiring a bandwidth of F hertz. How many channels can this system transmit simultaneously? Assuming all other factors to remain constant, show graphically the dependence of the number of channels that can be transmitted simultaneously on the operating wavelength of the system.
10. A signal is to be transmitted along a cable system of total length 125 km. The cable has an attenuation of 7 dB km^{-1} . Amplifiers, each having a gain of 43 dB, are placed at 6 km intervals along the cable
- State what is meant by the attenuation of a signal.
 - Calculate (i) the total attenuation caused by the transmission of the signal along the cable, (ii) the total signal gain as a result of amplification by all of the amplifiers along the cable.
 - The input signal has a power of 450 mW. Use your answers in (b) to calculate the output power of the signal as it leaves the cable system.



Answer & Solution

Q1.

Labelled element X represents the channel. Its function is to transmit information from one place to another.

Q2.

To avoid mixing up of signal from different transmitters the broadcast frequencies are sufficiently spaced in amplitude modulated wave.

This can be done by modulating the signals on high frequency carrier waves, e.g. frequency band for satellite communication is 5.925-6.425 GHz.

Q3

The power radiated by a linear antenna of length L is proportional to $(L/\lambda)^2$, where λ is the signal wavelength. Since for efficient transmission, the signal should be transmitted with high power, the signal should be of small wavelength or high frequency.

Q4

(i) **Transducer :**

Any device which converts one converts pressure, temperature, etc. into varying electrical signals I.e. transducer converts physical signals into electrical signals.

(ii) **Repeater :**

It picks up the signals from the transmitter, amplifies it and transmit it to the receiver. Thus, repeater comprises up of receiver, transmitter and amplifier. Its function is to extend the range of communication.

(iii) **Transmitter :**

It comprises of message signal source, modulator and transmitting antenna. Transmitter make signals compatible for communication channel *via* modulator and antenna.

(iv) **Bandpass filter :**

A device which passes the signals with certain frequency range only.

Q5

- (i) Modulation index, $\frac{E_m}{E_c} = \frac{10}{20} = 0.5$
- (ii) Side band frequencies = $f_c \pm f_m$
 $f_m = 10 \text{ kHz} \Rightarrow f_c = 1 \text{ MHz} = 1000 \text{ kHz}$
 \therefore Side band frequencies = 1000 ± 10
 $= 1010 \text{ kHz}, 990 \text{ kHz}$

Q6

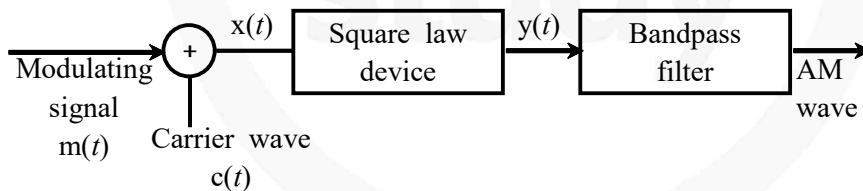
- (i) Line of Sight (LOS) is a type of propagation that can transmit & receive data only where transmit & receive stations are in view of each other without any sort of obstacle between them.

Ex – FM radio, Microwave & Satellite Transmission.

- (ii) The frequency of waves used for transmission of TV signals are of range 100 MHz-220 MHz. But ionosphere may be able to reflect waves back on earth of frequency up to 30 MHz. Therefore, ionosphere is unable to reflect TV waves (space waves) back on the earth.

Q7

Modulation is the process in which low frequency message signal is superimposed on high frequency carrier wave so that they can be transmitted over long distance. The block diagram for a simple modulator for obtaining AM signal is shown as below :



Q8

- (i) For amplitude modulation, message signal is used to modulate amplitude of a high frequency wave in input transistor of CE amplifier.

The output voltage is carrier signal varying in amplitude in accordance with biasing modulating voltage.

- (ii) Given, USB frequency = 660 kHz
 and LSB frequency = 640 kHz
 As USB = $f_c + f_m = 660 \text{ kHz}$
 and LSB = $f_c - f_m = 640 \text{ kHz}$
 $\therefore 2f_c = 660 + 640 = 1300$
 So, carrier frequency
 $f_c = 650 \text{ kHz}$
 and $2f_m = 20 \text{ kHz}$

$$\begin{aligned} \text{Bandwidth of frequencies required} \\ = \text{USB} - \text{LSB} = 660 - 640 = 20 \text{ kHz} \end{aligned}$$

Q9

Here, the wavelength of signal = λ

$$\text{Therefore, frequency of the signal, } v = \frac{c}{\lambda}$$

Since x % of the source frequency can be used as the bandwidth,

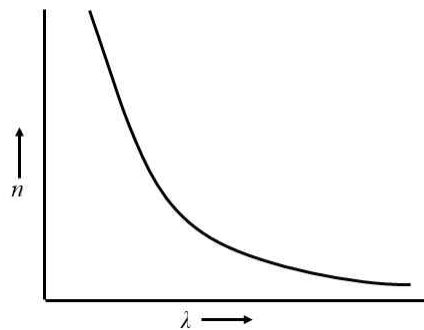
$$\text{available bandwidth} = \frac{v \times x}{100} = \frac{cx}{\lambda \times 100}$$

The bandwidth of the TV signal to be transmitted = F

Therefore, number of channels, the system can transmit,

$$\begin{aligned} n &= \frac{\text{available bandwidth}}{\text{bandwidth of TV signal}} \\ &= \frac{cx/\lambda \times 100}{F} = \frac{cx}{100\lambda F} \end{aligned}$$

As $n \propto 1/\lambda$, the dependence of number of channels on the operating wavelength of the system will be as shown in Fig. below.



Q10

(a) The loss of power in a signal, as it travels, is called attenuation.

(b) (i) Here, attenuation = 7 dB km^{-1}
length of the cable = 125 km
Hence, total attenuation in the cable
 $= 7 \times 125 = \mathbf{875 \text{ dB}}$

(ii) Since amplifiers are placed at 6 km intervals along the cable of length 125 km , it follows that in total 20 amplifiers will have to be placed.
Gain of each amplifier = 43 dB
Therefore, total signal gain = $20 \times 43 = \mathbf{860 \text{ dB}}$

(c) Here, power of the input signal,
 $P_1 = 450 \text{ mW} = 450 \times 10^{-3} \text{ W}$
The overall gain = $860 - 875 = 15 \text{ dB}$
If P_2 is the output power, then

$$\text{gain (or loss) in signal power} = 10 \log \frac{P_2}{P_1}$$

$$\text{or } -15 = 10 \log \frac{P_2}{450 \times 10^{-3}} \text{ or } \log \frac{P_2}{450 \times 10^{-3}} = -1.5$$

$$\text{or } \frac{P_2}{450 \times 10^{-3}} = 0.0316.$$

$$\text{or } P_2 = 0.0316 \times 450 \times 10^{-3} = 14 \times 10^{-3} = \mathbf{14 \text{ mW}}$$



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CBSE Exam Pattern Exercise Objective Questions (2)

- Which process is used in optical fibres
 - T.I.R
 - Scattering
 - Reflection
 - Dispersion
- An antenna behaves as a resonant circuit only when its length
 - equal $\frac{\lambda}{4}$
 - equal $\frac{\lambda}{2}$
 - equal $\frac{\lambda}{2}$ or its integral multiple
 - equal to 3λ
- In modulation process, radio frequency wave is termed as
 - modulating wave
 - modulated wave
 - carrier wave
 - modified wave
- A payload that is invariably found on all communication satellites is
 - optical telescope
 - camera
 - transponder
 - spectrometer
- Attenuation in optical fibre is mainly due to
 - Scattering
 - Dispersion
 - Absorption and scattering
 - Reflection



Answer & Solution

1. (a)
T.I.R
2. (c)
equal $\frac{\lambda}{2}$ or its integral multiple
3. (c)
carrier wave
4. (c)
transponder
5. (c)
Absorption and scattering

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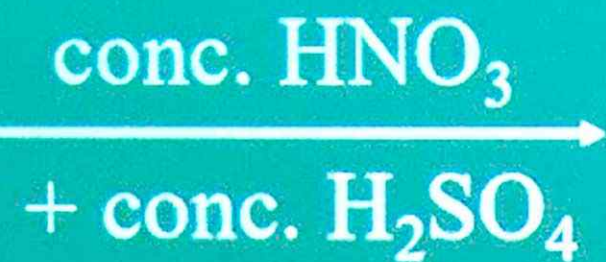
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Class 12 | Chemistry

02 Solution



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01. Introduction

When two or more chemically non-reacting substances are mixed and form homogeneous mixture it is called solution.

02. Types of Solution

	Solvent	Solute	Examples
1.	Gas	Gas	Mixture of gases, eg. air
2.	Gas	Liquid	Water vapour in air, mist. $\text{CHCl}_3(l) + \text{N}_2(g)$
3.	Gas	Solid	Smoke, camphor (s) + $\text{N}_2(g)$
4.	Liquid	Gas	CO_2 gas dissolve in water (aerated drink), soda water.
5.	Liquid	Liquid	Mixture of miscible liquids e.g. alcohol in water.
6.	Liquid	Solid	Salt in water, sugar in water.
7.	Solid	Gas	hydrogen over palladium.
8.	Solid	Liquid	Mercury in zinc, mercury in gold i.e. all amalgams.
9.	Solid	Solid	Alloys e.g. copper in gold. zinc in copper.

03. Mass Percentage

It may be defined as the number of parts of mass of solute per hundred parts by mass of solution.

$$\% \text{ by mass } \left(\frac{w}{W} \right) = \frac{\text{Wt. of solute}}{\text{Wt. of solution}} \times 100$$

[X % by mass means 100 gm solution contains X gm solute; (100–X) gm solvent]

04. Mass-Volume Percentage (W/V %) :

It may be defined as the mass of solute present in 100 cm³ of solution. For example, If 100 cm³ of solution contains 5 g of sodium hydroxide, then the mass-volume percentage will be 5% NaOH solution.

$$\% \left(\frac{w}{V} \right) = \frac{\text{wt. of solute (in gm)}}{\text{volume of solution (in mL)}} \times 100$$

[X% $\left(\frac{w}{V} \right)$ means 100 ml solution contains X gm solute]

05. Volume Percent

It can be represented as % v/v or % volume and used to prepare such solutions in which both components are in liquids state. It is the number of parts of by volume of solute per hundred parts by volume of solution

Therefore,

$$\% \left(\frac{v}{V} \right) = \frac{\text{volume of solute}}{\text{volume of solution}} \times 100$$

06. Parts Per Million (ppm)

This method is used for expressing the concentration of very dilute solutions such as hardness of water, air pollution etc.

$$\text{ppm of substance} = \frac{\text{Mass of solute} \times 10^6}{\text{Mass of solution}} = \frac{\text{Volume of solute} \times 10^6}{\text{Volume of solution}}$$

07. Mole Fraction:

The ratio of the number of moles of one component to the total number of all the components present in the solution, is called the mole fraction of that component.

Mole fraction of solute X_A is given by
$$X_A = \frac{n_A}{n_A + n_B} = \frac{n_A}{\sum n}$$

Mole fraction of solvent X_B is given by
$$X_B = \frac{n_B}{n_A + n_B} = \frac{n_B}{\sum n}$$

where n_A is moles of solute A and n_B is moles of solvent B.

For binary solution of A & B $X_A + X_B = 1$

08. Molarity (Molar Concentration) :

It is defined as the number of moles of the solute dissolved in per litre of the solution, i.e.,

$$\text{Molarity (M)} = \frac{\text{Number of moles of solute}}{\text{Volume of solution (in l)}} = \frac{w_A}{m_A \times V} = \frac{c(\text{gm/l})}{m_A} = \frac{\% \frac{W}{W} \times d \times 10}{m_A}$$

where let w_A g of the solute of molecular mass m_A be dissolved in V litre of solution, d = density of solution in g/mL.

09. Molarity of Dilute Solution :

$$\begin{array}{ccc} \text{Before dilution} & & \text{After dilution} \\ M_1V_1 & = & M_2V_2 \end{array}$$

Molarity of mixing :

Let there be three samples of solution (containing same solvent and solute) with their molarity M_1 , M_2 , M_3 and volumes V_1 , V_2 , V_3 respectively. These solutions are mixed; molarity of mixed solution may be given as:

$$M_1V_1 + M_2V_2 + M_3V_3 = M_R (V_1 + V_2 + V_3)$$

where, M_R = Resultant molarity

$V_1 + V_2 + V_3$ = Resultant volume after mixing

10. Some Important Point :

[Note : Molarity is dependent on volume, therefore, it depends on temperature.]

1 M	Molar solution, i.e., molarity is 1
0.5 M or M/2	Semimolar
0.1 M or M/10	Decimolar
0.01 M or M/100	Centimolar
0.001 M or M/1000	Millimolar

11. Molality (m) :

The number of moles or gram-mole of solute dissolve in 1000 gram of the solvent is called molality of the solution.

$$\text{Molality of a solution} = \frac{\text{Number of moles of solute}}{\text{Amount of solvent in kg.}} = \frac{\text{Number of moles of solute} \times 1000}{\text{Amount of solvent in grams.}}$$

12. Normality (N) :

The number of equivalents or gram equivalents of solute dissolved in one litre of the solution is known as normality (N) of the solution.

$$\text{Normality (N)} = \frac{\text{Number of gram equivalents of solute}}{\text{volume of solution in litre}} = \frac{\text{weight of solute in gram}}{\text{equivalent weight} \times \text{volume of sol}}$$

$$= \frac{\text{strength of solution in gram/litre}}{\text{Equivalent weight of solute}}$$

Equivalent weight of a substance is that weight which reacts with or displaces one gram of hydrogen, 8 grams of oxygen or 35.5 grams of chlorine.

13. Solubility of Gases

Gases dissolve in liquids in solids. For example, soda-water contains carbon dioxide dissolved in water under high pressure. Oxygen is sufficiently soluble in water to allow survival of aquatic life in lakes, rivers and oceans.

The solubility of a gas in a liquid is determined by several factors. In addition to the nature of the gas and the liquid, solubility of the gas depends on the temperature and pressure of the system. The solubility of a gas in a liquid is governed by Henry's Law which states that the solubility of a gas in a liquid is directly proportional to the pressure of the gas.

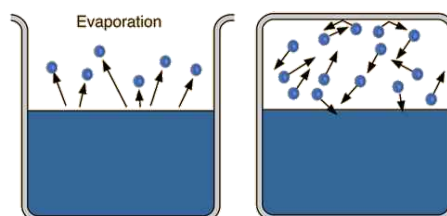
Mole fraction of the gas in a solution is proportional to the partial pressure of the gas. Or, partial pressure of the gas in solution = $K_H \times$ mole fraction of the gas in solution. Here K_H is Henry's law constant.

14. Vapour Pressure :

The Evaporation of a Liquid in a Closed Container

When a liquid is taken in a closed vessel at constant temperature, then there are two process which takes place.

- (i) **evaporation**
- (ii) **condensation**



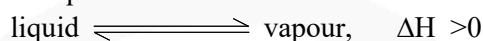
In the constant evaporation from the surface particles continue to break away from the surface of the liquid.

As the gaseous particles bounce around, some of them will hit the surface of the liquid again, and will be trapped there. This is called condensation. The rate of condensation increases with time, but rate of evaporation remain constant. There will rapidly be an equilibrium set up in which the number of particles leaving the surface is exactly balanced by the number rejoining it.

This pressure at equilibrium is called the saturated vapour pressure (also known as saturation vapour pressure).

15. Effect of Temperature on Vapour Pressure

When the space above the liquid is saturated with vapour particles, you have this equilibrium occurring on the surface of the liquid :



The forward change (liquid to vapour) is endothermic. It needs heat to convert the liquid into the vapour. According to Le Chatelier, increasing the temperature of a system in a dynamic equilibrium favours the endothermic change. That means increasing the temperature increase the amount of vapour present, and so increases the saturated vapour pressure.

16. Raoult's Law for Volatile Solute :

For a two component solution A (volatile) and B (volatile) the vapour pressure of solution is given by.

Vapour pressure of solution = Vapour pressure of solvent in solution + Vapour pressure of solute

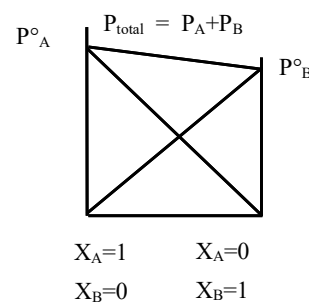
According to Raoult's Law partial. pressure of a component is equal to its mole fraction in solution multiplied with partial pressure in pure state

$$\text{Thus } P_A = P_A^\circ X_A$$

P_A° = vapour pressure in pure state of A

X_A = Mole fraction of A in solution

P_A = Partial pressure of A in solution



$$\text{Total vapour pressure of solution } P = P_A^\circ X_A + P_B^\circ X_B$$

17. Types of Solutions :

- (a) **Ideal Solution** : An ideal solution may be defined as the one which obeys Raoult's law over all concentration ranges at a given temperature. The total vapour pressure of an ideal solution containing liquids A and B is given by the following equation.

$$P = P_A + P_B = P_A^0 X_A + P_B^0 X_B$$

<i>Example</i>	benzene + toluene	chlorobenzene + bromobenzene
	ethyl bromide + ethyl iodide	n-butyl chloride + n-butyl bromide
	ethyl alcohol + methyl alcohol	

- (b) **Non-ideal solutions** : Solutions which do not over Raoult's law over all concentration ranges at constant temperature are called non-ideal solutions.

18. Distinction Between Ideal & Non Ideal Solutions

S.No.	Ideal Solution	Non Ideal Solution
1.	They obey Raoult's Law	They do not obey Raoult's Law
2.	$\Delta H_{\text{mixing}} = 0 \Rightarrow$ no heat is absorbed or released during dissolution	$\Delta H_{\text{mixing}} \neq 0 \Rightarrow$ no heat is absorbed or released during dissolution
3.	$\Delta V_{\text{mixing}} = 0 \Rightarrow$ the total volume of the solution is equal to the sum of the volume of the pure liquids mixed to form the solution	$\Delta V_{\text{mixing}} \neq 0 \Rightarrow$ the total volume of the solution is not equal to the sum of the volume of the pure liquids mixed to form the solution
4.	In ideal solution $P_A = P_A^0 X_A$	In non ideal solution $P_A \neq P_A^0 X_A$
5.	Components of ideal solution can be separated in pure form by fractional distillation	Components of non ideal solution can not be separated in pure form by fractional distillation

19. Types of Non - Ideal Solutions

- (I) Non ideal solutions showing positive deviation.
(II) Non ideal solutions showing negative deviation.

20. Non Ideal Solutions Showing Positive Deviation

Condition for forming non-ideal solution showing positive deviation from Raoult's law. Two liquids A and B on mixing form this type of solution when

- A—B attractive force should be weaker than A—A and B—B attractive forces.
- 'A' and 'B' have different shape, size and character.
- 'A' and 'B' escape easily showing higher vapour pressure than the expected value.

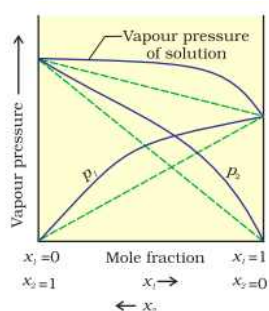
Example

acetone + ethanol
water + ethanol
acetone + benzene

acetone + CS₂
CCl₄ + toluene
CCl₄ + CH₃OH

water + methanol
CCl₄ + CHCl₃
cyclohexane + ethanol

21. Graphical Representation of Vapour Pressure of Non-Ideal Solution Showing Positive Deviation



22. Non Ideal Solutions Showing Negative Deviation.

Condition for forming non-ideal solution showing negative deviation from Raoult's law.

Two liquids A and B on mixing form this type of solution when

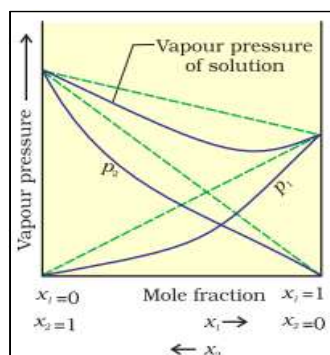
- A—B attractive force should be greater than A—A and B—B attractive forces.
- 'A' and 'B' have different shape, size and character.
- Escaping tendency of both components 'A' and 'B' is lowered showing lower vapour pressure than expected ideally.

Example

acetone + aniline;
chloroform + diethyl ether,
chloroform + benzene
water + HCl

CH₃OH + CH₃COOH;
acetic acid + pyridine;
H₂O + HNO₃;

23. Graphical Representation of Vapour of non-ideal Solution Showing Negative Deviation



24. Distinction Between Non Ideal Solutions Showing Positive Deviation & Negative Deviation

S.NO.	Showing positive deviation	Showing negative deviation
1.	$\Delta H_{\text{mix}} > 0$. (endothermic dissolution i.e. heat is absorbed)	$\Delta H_{\text{mix}} < 0$. (Exothermic dissolution i.e. heat is evolved)
2.	$\Delta V_{\text{mix}} > 0$. (Volume is increased after dissolution)	$\Delta V_{\text{mix}} < 0$. (Volume is decreased after dissolution)
3.	$p_A > p_A^0 X_A$; $p_B > p_B^0 X_B$, $\therefore p_A + p_B > p_A^0 X_A + p_B^0 X_B$	$p_A < p_A^0 X_A$; $p_B < p_B^0 X_B$, $\therefore p_A + p_B < p_A^0 X_A + p_B^0 X_B$

25. Colligative Properties

The properties of dilute solutions containing nonvolatile solute, which depends upon relative number of solute and solvent particles but do not depend upon their nature are called colligative properties.

Some of the colligative properties are

- (i) Relative lowering of vapour pressure
- (ii) Elevation in boiling point
- (iii) Depression in freezing point and
- (iv) Osmotic pressure.

26. Factor that Affect the Colligative Property

The number of solute particles in solution. To be more accurate, the colligative property depends upon the fraction of solute and solvent particles in solution.

- (1) Nature of the solvent
- (2) Independent of the nature of the solute
- (3) Extent of association and dissociation of solute particles in solution.

27. Relative Lowering of Vapour Pressure

As shown earlier the mathematical expression for relative lowering of vapour pressure is as follows

$$\frac{\Delta P}{P_A^0} = X_B = \text{mole fraction of solute}$$

$\Delta P = P_A^0 - P_A = \text{lowering of vapour pressure}$

$P_A = \text{vapour pressure of pure solvent}$

Molecular mass of non-volatile substance can be determined from relative lowering of vapour pressure

$$\frac{P_A^0 - P}{P_A^0} = \frac{W_B/M_B}{W_A/M_A + W_B/M_B}$$

For dilute solution $W_B/M_B < W_A/M_A$ and hence W_B/M_B may be neglected in the denominator. So

$$\frac{P_A^0 - P_A}{P_A^0} = \frac{W_B/M_B}{W_A/M_A} = \frac{W_B}{W_A} \times \frac{M_A}{M_B} \text{ or } M_B = \frac{W_B/M_A}{W_A} \left(\frac{P_A^0}{P_A^0 - P_A} \right)$$

28. Elevation in Boiling Point

The boiling points elevates when a non-volatile solute is added to a volatile solvent. Which occurs due to lowering of vapour pressure. The boiling point of a liquid may be defined as the temperature at which its vapour pressure becomes equal to atmospheric pressure.

So when a non-volatile solute is added to a volatile solvent results lowering of vapour pressure and consequent elevation of boiling point. where

$$\Delta T_b = T_b - T_b^0$$

$\Delta T_b = \text{elevation in B.P.}$

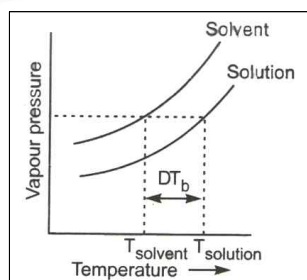
$\Delta P = \text{lowering of V.P.}$

$X_B = \text{mole fraction of solute}$

$K = \text{elevation constant}$

$T_b^0 = \text{boiling point of solvent}$

$T_b = \text{boiling point of solution}$



It is found that elevation of boiling point is directly proportional to the number of moles of the solute in given amount of the solvent(m).

$$\Delta T_b \propto m$$

$$\Delta T_b \propto K_b m$$

Where 'm' is the molality of solution.

Where K_b is ebullioscopic or boiling point elevation constant. When molality of the solution is equal to one. Then

$$\Delta T_b = K_b$$

Determination of K_b of solvent:

$$K_b = \frac{RT_b^2}{1000L_v}$$

where R is molar gas constant, T_b is the boiling point of the solvent on Kelvin scale and L_v the latent heat of vaporization of solvent in calories per gram.

$$\text{For water} \quad K_b = \frac{2 \times (373)^2}{1000 \times 540} = 0.515 \text{ K}\cdot\text{kg/mol}$$

29. Depression in Freezing Point

The freezing point of a pure liquid is fixed. If a non-volatile solute is dissolved in a liquid the freezing point of the solution is lowered. The freezing point is that temperature at which the solvent has the same vapour pressure in two phases liquid solution and solid solvent. Since the solvent vapour pressure in solution is depressed. Its vapour pressure will become equal to that of the solid solvent at a lower temperature.

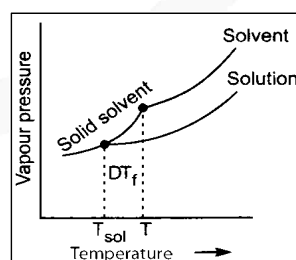
$$\Delta T_f = T_f^0 - T_f$$

It is found that depression in freezing point is directly proportional to the number of moles of the solute in given amount of the solvent(m).

Hence $\Delta T_f \propto m$

$$\Delta T_f \propto K_f m$$

Where m = molarity of the solution.



K_f = molal depression constant

When molarity (m) of the solution is one. then

$$\Delta T_f = K_f$$

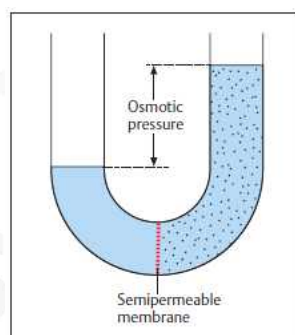
Hence molal depression constant or molal cryoscopic constant may be defined as “the depression in freezing point when one mole of non-volatile solute is dissolved per kilogram (1000 gm) of solvent” and molal depression constant is defined as “the depression in freezing point when one mole of non-volatile solute is dissolved per litre (1000 ml) of solvent.” The molecular mass of the non-volatile solute may be calculated by the use of following mathematical equation

$$M_B = \frac{K_f \times W_B \times 1000}{W_A \times \Delta T_f}$$

Where W_A = mass of solvent, W_B = mass of solute,
 M_A = Molar mass of solvent, M_B = Molar mass of solute.

30. Osmotic Pressure

- (i) **Osmosis** : Spontaneous flow of solvent molecules through a semipermeable membrane from a pure solvent to the solution (or from a dilute solution to a concentrated solution) is termed as osmosis.



- (ii) **Osmotic Pressure** :

When a solution is separated from the pure solvent with the help of a semipermeable membrane. There occurs the flow of solvent molecules from the pure solvent to the solution side. The flow of solvent molecules from a region of higher concentration of solvent to the region of lower concentration of solvent is termed as the phenomenon of osmosis. This also happens when two solution of different concentrations are separated by a semipermeable membrane.

The excess pressure that must be applied to the solution side to prevent the passage of solvent into it through a semipermeable membrane.

- (iii) **Theory of Dilute Solutions** :

The osmotic pressure of dilute solution was the same as the solute would have exerted if it were a gas at the same temperature as of the solution and occupied a volume equal to that of the solution. This generalization is known as Van't Hoff theory of dilute solutions. The osmotic pressure is a colligative property. So the osmotic pressure is related to the number of moles of the solute by the following relation

$$\pi V = nRT$$

$$\pi = \frac{n}{V} RT \quad (\because \frac{n}{V} = C)$$

$$\pi = CRT$$

Here C = concentration of solution in moles per litre;

R = solution constant;

T = temperature in Kelvin degree;

n = number of moles of solute; and

V = volume of solution.

This equation is called Van't Hoff's equation.

31. Type of Solution :

(i) **Isotonic solution :**

The two solutions having equal osmotic pressure are termed as isotonic solution.

(ii) **Hypertonic solution :**

A solution having higher osmotic pressure than some other solution is said to be called hypertonic solution.

(iii) **Hypotonic solution :**

A solution having a lower osmotic pressure relative to some other solution is called hypotonic solution.

Semipermeable membrane :

A membrane which allows the passage of solvent molecules but not that of solute. When a solution is separated from the solvent by it is known as semipermeable membrane.

Some example of it are as follows

(a) Copper ferrocyanide $\text{Cu}_2[\text{Fe}(\text{CN})_6]$

(b) Calcium phosphate membrane and

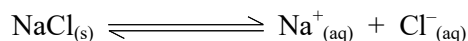
(c) Phenol saturated with water.

Reverse Osmosis :

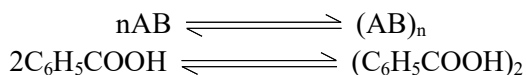
If a pressure greater than the osmotic pressure is applied on the concentrated solution, the solvent starts to flow concentrated solution to dilute solution (or pure solvent). This is reverse osmosis. One of its chief uses is desalination of sea water to get pure drinking water.

32. Abnormal Molecular Masses

Vapour pressure lowering, boiling point elevation, freezing point depression and osmotic pressure are colligative properties which depend upon the fraction of solute and solvent particles in solution and not upon the chemical nature of the solute. If solute molecules dissociates in solution, there are more particles in solution and therefore, lowering of vapour pressure shows an increased effect.



If the solute molecules associates in solution, there are less particles in solution, and therefore lowering of vapour pressure shows a decreased effect.



33. Vant Hoff Factor


In 1886, Van't Hoff, Jacobs Henricus (Dutch chemist, 1859,-1911) introduced a factor 'i' known as Van't Hoff factor to express the extent to association or dissociation of a solute in solution. It can be calculated as:

$$\begin{aligned} i &= \frac{\text{number of solute particles actually present in solution}}{\text{number of solute particles dissolved}} \\ &= \frac{\text{Observed colligative property}}{\text{normal colligative property}} = \frac{\text{observed molality}}{\text{normal molality}} = \frac{\text{normal molecular weight of solute}}{\text{observed molecular weight of solute}} \end{aligned}$$

The Van't Hoff factor for a solute can be calculated by the following modified equations:

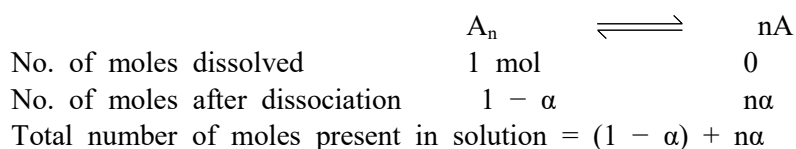
$$\begin{aligned} \text{(i)} \quad \frac{P_A^\circ - P_A}{P_A^\circ} &= iX_B & \text{(iii)} \quad \Delta T_b &= iK_b m \\ \text{(ii)} \quad \Delta T_f &= iK_f m & \text{(iv)} \quad \pi &= iCRT \end{aligned}$$

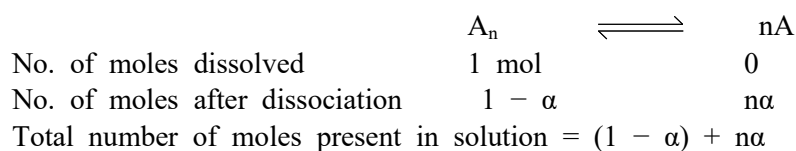
where C is molarity of the solution.

NOTE  For non-electrolytes; $i = 1$
For electrolytes; $i > 1$ (If solute particles undergo Dissociation in the solution)
 $i < 1$ (If solute particles undergo Association in the solution)

34. Application of Van't Hoff Factor :

(a) Calculation of Degree of Dissociation of solute particles:



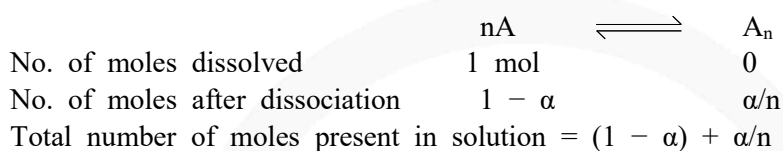
(b) Calculation of Degree of Dissociation of solute particles:

$$\text{Van't Hoff factor, } i = \frac{\text{Moles of solute actually present in solution}}{\text{Moles of solute dissolved}}$$

$$= \frac{(1-\alpha)+n\alpha}{1} = 1+(n-1)\alpha \quad \text{or} \quad \alpha = \frac{i-1}{n-1}$$

(c) Calculation of Degree of Association of solute particles:

Let n moles of the solute, A , associate to form $(A)_n$. If α is the degree of association.



$$i = \left[1 - \alpha \left(1 - \frac{1}{n} \right) \right] \quad \text{Hence } \alpha = \frac{i-1}{\frac{1}{n}-1} = (i-1) \times \frac{n}{1-n}$$

CBSE Exam Pattern Exercise

Subjective Questions (1)

(Q 1 to 2) One Mark

1. What role does the molecular interaction play in solution of alcohol and water?
2. Why do gases nearly always tend to be less soluble in liquid as the temperature is raised?

(Q 3 to 4) Two Marks

3. If the solubility product of CuS is 6×10^{-16} , calculate the maximum molarity of CuS in aqueous solution.
4. State Henry's law and mention some of its important applications.

(Q 5 to 6) Three Marks

5.
 - (a) Determine the amount of CaCl_2 ($i = 2.47$) dissolved in 2.5 litre of water such that its osmotic pressure is 0.75 atm at 27°C .
 - (b) Amongst the following compounds, identify which are insoluble, partially soluble and highly soluble in water (i) phenol, (ii) toluene, (iii) formic acid, (iv) ethylene glycol, (v) chloroform, (vi) pentanol
6. Calculate the mass of a non-volatile solute (molar mass 40 g mol^{-1}) which should be dissolved in 114g octane to reduce its vapour pressure to 80%.

(Q 7 to 8) Four Marks

7. 100 g of liquid A (molar mass 140 g mol^{-1}) was dissolved in 1000 g of liquid B (molar mass 180 g mol^{-1}). The vapour pressure of pure liquid B was found to be 500 torr. Calculate the vapour pressure of pure liquid A and its vapour pressure in the solution if the total vapour pressure of the solution is 475 torr.
8. The air is a mixture of a number of gases. The major components are oxygen and nitrogen with approximate proportion of 20% is to 79% by volume at 298 K. The water is in equilibrium with air at a pressure of 10 atm. At 298 K, if the Henry's law constants for oxygen and nitrogen are $3.30 \times 10^7 \text{ mm}$ and $6.51 \times 10^7 \text{ mm}$ respectively, calculate the composition of these gases in water.

(Q 9 to 10) Five Marks

9. Calculate the depression in the freezing point of water when 10 g of $\text{CH}_3\text{CH}_2\text{CHClCOOH}$ is added to 250 g of water. $K_a = 1.4 \times 10^{-3}$, $K_f = 1.86 \text{ K kg mol}^{-1}$.
10. 19.5g of CH_2FCOOH is dissolved in 500g of water. The depression in the freezing point observed is 1.0°C . Calculate the van't hof factor and dissociation constant of fluoroacetic acid. K_f for water is $1.86 \text{ K kg mol}^{-1}$.



Answer & Solution

Q1

There is strong hydrogen bonding in alcohol molecules as well as water molecules. On mixing, the molecular interactions are weakened. Hence, their solution will show positive deviations from ideal behaviour. As a result, the solution will have higher vapour pressure and lower boiling point than that of water and alcohol.

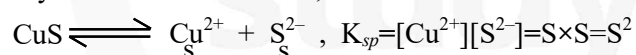
Q2

Dissolution of gas in liquid is an exothermic process ($\text{Gas} + \text{Solvent} \rightleftharpoons \text{Solvent} + \text{Heat}$). As the temperature is increased, equilibrium shifts backward.

Q3

Maximum molarity of CuS in aqueous solution = Solubility of CuS in mol L^{-1}

If S is the solubility of CuS in mol L^{-1} , then



$$\therefore S^2 = 6 \times 10^{-6} \text{ or } S = \sqrt{6 \times 10^{-6}} = 2.45 \times 10^{-3} \text{ mol L}^{-1}.$$

Q4

The mass of a gas dissolved in a given volume of the liquid at constant temperature is directly proportional to the pressure of the gas present in equilibrium with the liquid.

Mathematically, $m \propto P$ or $m = Kp$

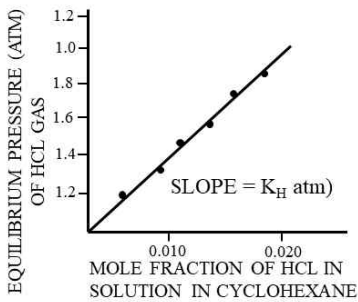
Or

The solubility of a gas in a liquid at a particular temperature is directly proportional to the pressure of the gas in equilibrium with the liquid at that temperature.

$$\text{From eqn (ii), } p_A = \frac{1}{K} x_A \text{ or } P_A = K_H x_A$$

The partial pressure of a gas in vapour phase (p) is directly proportional to the mole fraction (x) of the gas in the solution.

02 Solution



Plot of mole fraction in solution versus equilibrium pressure

Application of Henry's law:

- (i) In the production of carbonated beverages.
- (ii) In the deep sea diving.
bend or decompression sickness
- (iii) In the function of lungs.
- (iv) For climbers or people living at high altitudes.

Q5

$$\pi = i CRT = i \frac{n}{V} RT \quad \text{or} \quad n = \frac{\pi \times V}{i \times R \times T} = \frac{0.75 \text{ atm} \times 2.5 \text{ L}}{2.47 \times 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1} \times 300 \text{ K}} = 0.0308 \text{ mole}$$

Molar mass of $\text{CaCl}_2 = 40 + 2 \times 35.5 = 111 \text{ g mol}^{-1} \therefore$ Amount dissolved = $0.0308 \times 111 \text{ g} = 3.42 \text{ g}$

Q6

Reduction of vapour pressure to 80% means that if $p^\circ = 100 \text{ mm}$, then $p_s = 80 \text{ mm}$. Applying complete formula

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n_2}{n_1 + n_2} = \frac{w_2 M_2}{w_1 / M_1 + w_2 / M_2}$$

$$\frac{100 - 80}{100} = \frac{w_2 / 40}{114 / 114 + w_2 / 40} \quad (\text{Mol. mass of octane } \text{C}_8\text{H}_{18} = 114 \text{ g mol}^{-1})$$

$$\text{or} \quad \frac{20}{100} = \frac{w_2 / 40}{1 + w_2 / 40} \quad \text{or} \quad \frac{1}{5} \left(1 + \frac{w_2}{40} \right) = \frac{w_2}{40} \quad \text{or} \quad w_2 = 10 \text{ g}$$

Note that complete formula is required because concentration of solution is greater than 5%

Complete formula can also be applied in the form

$$\frac{p^\circ - p_s}{p_s} = \frac{w_2 / M_2}{w_1 / M_1} \quad \text{or} \quad \frac{100 - 80}{80} = \frac{w_2 / 40}{114 / 114} \quad \text{or} \quad \frac{1}{4} = \frac{w_2}{40} \quad \text{or} \quad w_2 = 10 \text{ g}$$

Alternatively, suppose mass of solute dissolved = $w \text{ g}$

$$\text{Moles of solute} = \frac{w}{40} \text{ g}$$

$$\text{Moles of solvent (octane)} = \frac{114}{114} = 1 \text{ mole} \quad (\text{Mol. mass of } \text{C}_8\text{H}_{18} = 114 \text{ g mol}^{-1})$$

$$\therefore \text{Mole fraction of solvent} = \frac{1}{1 + w/40}$$

For a non-volatile solute,

Vapour pressure of solution = Mole fraction of solvent in the solution \times Vapour pressure of pure

02 Solution

solvent

$$P_s = x_i \times p^\circ$$

$$\therefore 80 = \frac{1}{1 + w/40} \times 100 \text{ or } 1 + \frac{w}{40} = \frac{100}{80} \text{ or } \frac{w}{40} = \frac{10}{8} - 1 = \frac{2}{8} = \frac{1}{4} \text{ or } w = 10\text{g}$$

Q7

$$\text{No of moles of liquid A (solute)} = \frac{100\text{g}}{140\text{g mol}^{-1}} = \frac{5}{7} \text{ mole}$$

$$\text{No. of moles of liquid B (solvent)} = \frac{1000\text{g}}{180\text{g mol}^{-1}} = \frac{50}{9} \text{ mole}$$

$$\therefore \text{Mole fraction of A in the solution } (x_A) = \frac{5/7}{5/7 + 50/9} = \frac{5/7}{395/63} = \frac{5}{7} \times \frac{63}{395} = \frac{45}{395} = 0.114$$

$$\therefore \text{Mole fraction of B in the solution } (x_B) = 1 - 0.114 = 0.886$$

Also, given $P_B^\circ = 500 \text{ torr}$

Applying Raoult's law, $P_A = x_A P_A^\circ = 0.114 \times P_A^\circ$

$$P_B = x_B P_B^\circ = 0.886 \times 500 = 443 \text{ torr}$$

$$P_{\text{Total}} = P_A + P_B$$

$$475 = 0.114 P_B^\circ + 443 \text{ or } P_A^\circ = \frac{475 - 443}{0.114} = 280.7 \text{ torr}$$

$$\text{Substituting this value in eqn. (i), we get } P_A = 0.114 \times 280.7 \text{ torr} = 32 \text{ torr.}$$

Q8

Total pressure of air in equilibrium with water = 10 atm

As air contains 20% oxygen and 79% nitrogen by volume,

$$\therefore \text{Partial pressure of oxygen } (p_{O_2}) = \frac{20}{100} \times 10 \text{ atm} = 2 \text{ atm} = 2 \times 760 \text{ mm} = 1520 \text{ mm}$$

$$\text{Partial pressure of nitrogen } (p_{N_2}) = \frac{79}{100} \times 10 \text{ atm} = 7.9 \text{ atm} = 7.9 \times 760 \text{ mm} = 6004 \text{ mm}$$

$$K_H(O_2) = 3.30 \times 10^7 \text{ mm}, K_H(N_2) = 6.51 \times 10^7 \text{ mm}$$

$$\text{Applying Henry's law, } P_{O_2} = K_H \times x_{O_2} \text{ or } x_{O_2} = \frac{P_{O_2}}{K_H} = \frac{1520 \text{ mm}}{3.30 \times 10^7 \text{ mm}} = 4.61 \times 10^{-5}$$

$$p_{N_2} = K_H \times x_{N_2} \text{ or } x_{N_2} = \frac{P_{N_2}}{K_H} = \frac{6004 \text{ mm}}{6.51 \times 10^7 \text{ mm}} = 9.22 \times 10^{-5}$$

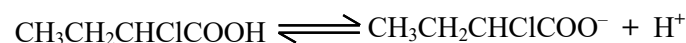
Q9

Molar mass of $\text{CH}_3\text{CH}_2\text{CHClCOOH} = 15 + 14 + 13 + 35.5 + 45 = 122.5 \text{ g mol}^{-1}$

$$10 \text{ g of } \text{CH}_3\text{CH}_2\text{CHClCOOH} = \frac{10}{122.5} \text{ mole} = 8.16 \times 10^{-2} \text{ mole}$$

$$\therefore \text{Molality of the solution } (m) = \frac{8.16 \times 10^{-2} \text{ mole}}{250 \text{ g}} \times 1000 \text{ g kg}^{-1} = 0.3264$$

Of α is the degree of dissociation of $\text{CH}_3\text{CH}_2\text{CHClCOOH}$, then



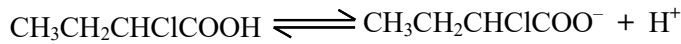
Initial conc.	C	mol L ⁻¹	0	0
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02 Solution

At eqm. $C(1-\alpha)$ $C\alpha$ $C\alpha$

$$\therefore K_a = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)} \approx C\alpha^2 \quad \text{or} \quad \alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1.4 \times 10^{-3}}{0.3264}} = 0.065$$

To calculate van't Hoff factor:



Initial moles 1

Moles at eqm. $1 - \alpha$ α α $\text{Total} = 1 + \alpha$

$$i = \frac{1 + \alpha}{1} = 1 + \alpha = 1 + 0.065 = 1.065 ; \Delta T_f = i K_f m = (1.065)(1.86)(0.3264) = \mathbf{0.65^\circ}$$

Q10

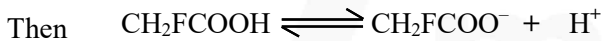
Hence, $W_2 = 19.5\text{g}$, $w_1 = 500\text{g}$, $K_f = 1.86\text{K kg mol}^{-1}$, $(\Delta T_f)_{\text{obs}} = 1.0^\circ$

$$\therefore M_2 (\text{observed}) = \frac{100K_f w_2}{w_1 \Delta T_f} = \frac{(1000\text{g kg}^{-1})(1.86\text{K kg mol}^{-1})(19.5\text{g})}{(500\text{g})(1.0\text{K})} = 72.54\text{g}$$

M_2 (calculated) for $\text{CH}_2\text{FCOOH} = 14 + 19 + 45 = 78\text{g mol}^{-1}$

$$\text{van't Hoff factor } (i) = \frac{(M_2)_{\text{cal}}}{(M_2)_{\text{obs}}} = \frac{78}{72.54} = \mathbf{1.0753}$$

Calculation of dissociation constant. Suppose degree of dissociation at the given concentration is α .



Initial $C\text{ mol L}^{-1}$ 0 0

At eqm. $C(1-\alpha)$ $C\alpha$ $C\alpha$, $\text{Total} = C(1+\alpha)$

$$i = \frac{C(1+\alpha)}{C} = 1 + \alpha \quad \text{or} \quad \alpha = i - 1 = 1.0753 - 1 = 0.0753$$

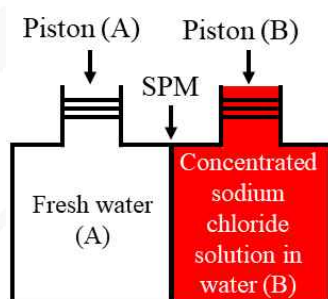
$$K_a = \frac{[\text{CH}_2\text{FCOO}^-][\text{H}^+]}{[\text{CH}_2\text{FCOOH}]} = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)} = \frac{C\alpha^2}{1-\alpha}$$

Taking volume of the solution as 500 mL,

$$C = \frac{19.5}{78} \times \frac{1}{500} \times 1000 = 0.5\text{M} \quad \therefore K_a = \frac{C\alpha^2}{1-\alpha} = \frac{(0.5)(0.0753)^2}{1-0.0753} = \mathbf{3.07 \times 10^{-3}}$$

CBSE Exam Pattern Exercise Objective Questions (2)

- Considering the formation, breaking and strength of hydrogen bond, predict which of the following mixtures will show a positive deviation from Raoult's law ?
 - Methanol and acetone
 - Chloroform and acetone
 - Nitric acid and water
 - Phenol and aniline
- Consider the figure and mark the correct option.
 - water will move from side (A) to side (B) if a pressure lower than osmotic pressure is applied on piston (B)



- water will move from side (B) to side (A) if a pressure greater than osmotic pressure is applied on piston (B)
 - water will move from side (B) to side (A) if a pressure equal to osmotic pressure is applied on piston (B)
 - water will move from side (A) to side (B) if pressure equal to osmotic pressure is applied on piston (A)
- If two liquids A and B form minimum boiling azeotrope at some specific composition, then
 - A–B interactions are stronger than those between A–A or B–B
 - vapour pressure of solution increases because more number of molecules of liquids A and B can escape from the solution
 - vapour pressure of solution decreases because less number of molecules of only one of the liquids escape from the solution
 - A–B interactions are weaker than those between A–A or B–B
 - 4L of 0.02 M aqueous solution of NaCl was diluted by adding one litre of water. The molality of the resultant solution is
 - 000.4
 - 0.008
 - 0.012
 - 0.016

5. K_H value for Ar(g), CO₂(g), HCHO (g) and CH₄(g) are 40.39, 1.67, 1.83×10^{-5} and 0.413 respectively.

Arrange these gases in the order of their increasing solubility.

- (a) HCHO < CH₄ < CO₂ < Ar
- (b) HCHO < CO₂ < CH₄ < Ar
- (c) Ar < CO₂ < CH₄ < HCHO
- (d) Ar < CH₄ < CO₂ < HCHO

The questions given below consists of an Assertion and the Reason. Use the following key to choose the appropriate answer.

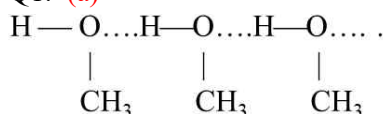
- (a) If both assertion and reason are CORRECT and reason is the CORRECT explanation of the assertion.
 - (b) If both assertion and reason are CORRECT, but reason is NOT THE CORRECT explanation of the Assertion.
 - (c) If assertion is CORRECT, but the reason is INCORRECT.
 - (d) If both assertion and reason are INCORRECT.
6. **Assertion** : When NaCl is added to water, a depression in freezing point is observed.
Reason : The lowering of vapour pressure of a solution causes depression in the freezing point.

miso
study



Answer & Solution

Q1. (a)



On adding acetone, its molecules get in between the molecules of methanol breaking hydrogen bonds and reducing methanol-methanol attractions.

Q2. (b)

The process represented in the image is the reverse osmosis (R.O) which takes place when pressure more than osmotic pressure is applied to the solution.

Due to this, the solvent will flow from the solution into the pure solvent through semi permeable membrane.

Q3. (d)

Minimum boiling azeotrope is formed when actual vapour pressure is higher than expected, i.e. solution shows +ve deviation from Raoult's law which is so when A-B interactions are weaker than A-A or B-B interactions.

Q4. (d)

$$M_1V_1 = M_2V_2, 0.02 \times 4 = M_2 \times 5 \text{ or } M_2 = 0.016$$

Q5. (c)

acceleration to the henry's law

$$P_A = K_H x_A$$

If x_A has a lower value, then K_H will increase, \therefore solubility decrease.

Q6. (a)

Reason is the correct explanation of the assertion.

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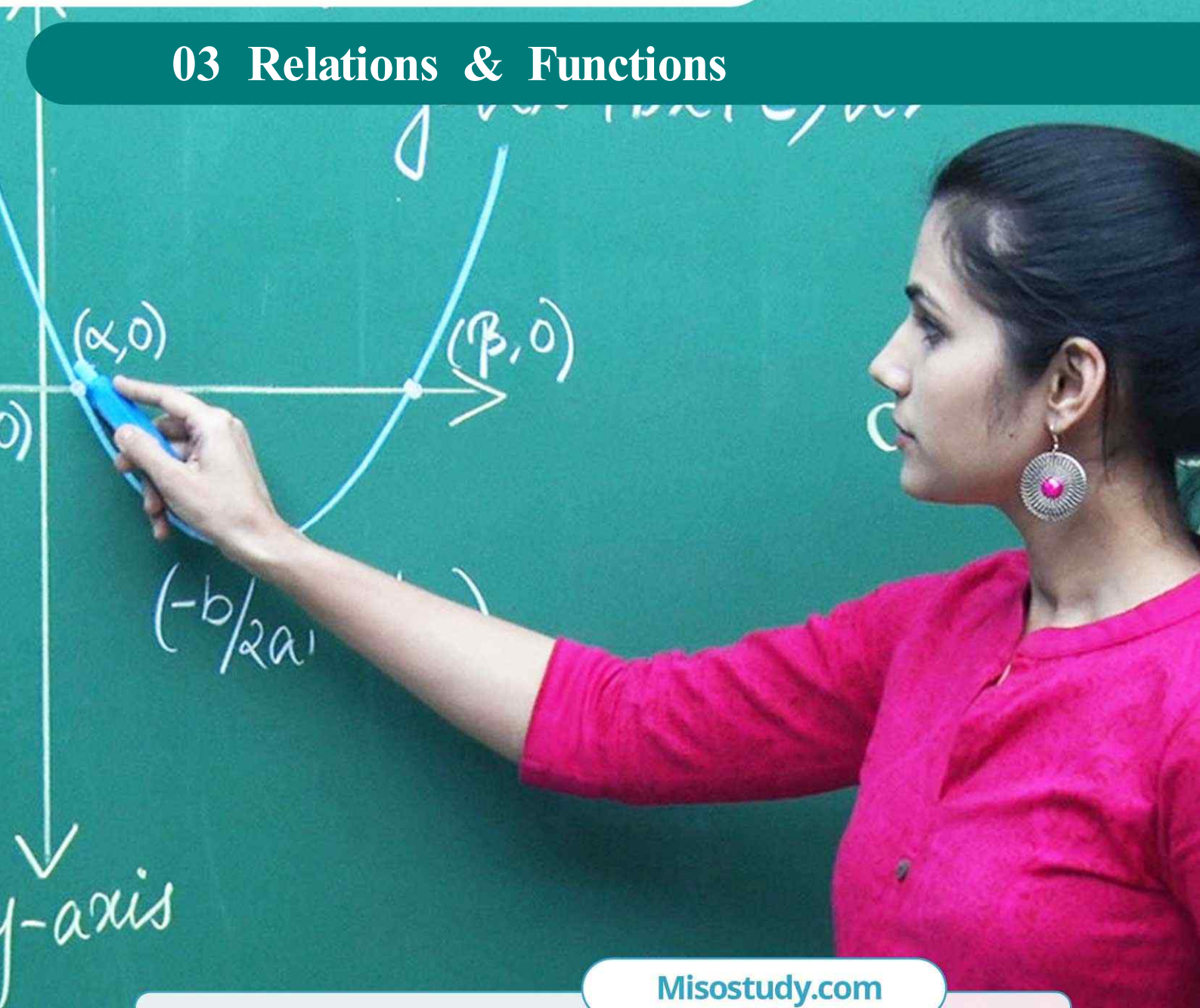
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03 Relations & Functions



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01. Types of Relations


(A) VOID, UNIVERSAL AND IDENTITY RELATIONS

Void Relation-

Let A be a set. Then, $\phi \subseteq A \times A$ and so it is a relation on A . This relation is called the void or empty relation on set A .

Universal Relation-

Let A be a set. Then, $A \times A \subseteq A \times A$ and so it is a relation on A . This relation is called the universal relation on A .

NOTE  It is to note here that the void relation and the universal relation on a set A are respectively the smallest and the largest relations on set A . Both the empty (or void) relation and the universal relation are sometimes. They are called trivial relations.

Identity Relation-

Let A be a set. Then, the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A .

In other words, a relation I_A on A is called the identity relation i.e., if every element of A is related to itself only.

(B) REFLEXIVE, SYMMETRIC, TRANSITIVE RELATIONS

Reflexive Relation-

A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.

A relation R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

Illustration I Let $A = \{1, 2, 3\}$ be a set. Then $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)\}$ is a reflexive relation on A . But $R_1 = \{(1, 1), (3, 3), (2, 1), (3, 2)\}$ is not a reflexive relation on A , because $2 \in A$ but $(2, 2) \notin R_1$.


Symmetric Relation-

A relation R on a set A is said to be a symmetric relation iff

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$.

Illustration I Let $A = \{1, 2, 3, 4\}$ and let R_1 and R_2 be relations on A given by $R_1 = \{(1, 3), (1, 4), (3, 1), (2, 2), (4, 1)\}$ and $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Clearly, R_1 is a symmetric relation on A . However, R_2 is not $\in R_2$ but $(3, 1) \notin R_2$.

NOTE  A reflexive relation on a set A is not necessarily symmetric. For example, the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ is a reflexive relation on set $A = \{1, 2, 3\}$ but it is not symmetric.

Transitive Relation-

Let A be any set. A relation R on A is said to be a transitive relation iff

$$(a, b) \in R \text{ and } (b, c) \in R$$

$$\Rightarrow (a, c) \in R \text{ for all } a, b, c \in A.$$

$$\text{i.e. } aRb \text{ and } bRc$$

$$\Rightarrow aRc \text{ for all } a, b, c \in A.$$

Illustration I On the set N of natural numbers, the relation R defined by $xRy \Rightarrow x$ is less than y is transitive, because for any $x, y, z \in N$
 $x < y$ and $y < z \Rightarrow x < z$ i.e., xRy and $yRz \Rightarrow xRz$

(C) Equivalence Relation

A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Let R be a relation on the set of all line in a plane defined by $(l_1, l_2) \in R \Leftrightarrow$ line l_1 is parallel to line l_2 .

Then, R is an equivalence relation.

Solution : Let L be the given set of all lines in a plane. Then we observe the following properties.

Reflexive : For each line $l \in L$, we have

$$l \parallel l \Rightarrow (l, l) \in R \text{ for all } l \in L \Rightarrow R \text{ is reflexive}$$

Symmetric : Let $l_1, l_2 \in L$ such that $(l_1, l_2) \in R$. Then,

$$(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R. \text{ So, } R \text{ is symmetric on } R.$$

Transitive : Let $l_1, l_2, l_3 \in L$ such that $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$. Then,

$$(l_1, l_2) \in R \text{ and } (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2 \text{ and } l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$$

So, R is transitive on L , Hence R being reflexive symmetric and transitive is an equivalence relation on L .

02. Kinds of Functions

ONE-ONE FUNCTION (INJECTION)

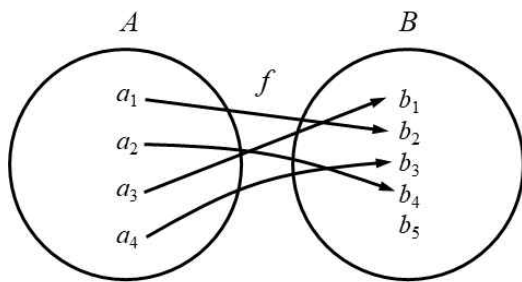
A function $f : A \rightarrow B$ is said to be a one-one function or an injection if different elements of A have different images in B .

Thus, $f : A \rightarrow B$ is one-one

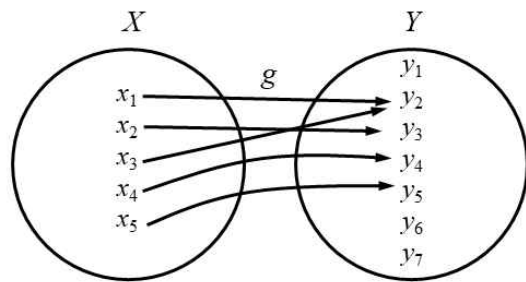
$$\Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b) \text{ for all } a, b \in A$$

$$\Leftrightarrow f(a) = f(b) \Rightarrow a = b \text{ for all } a, b \in A$$

Example : Let $f : A \rightarrow B$ and $g : X \rightarrow Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, $f : A \rightarrow B$ is a one-one function. But, $g : X \rightarrow Y$ is not one-one because two distinct elements x_1 and x_3 have the same image under function g .

Let $f : A \rightarrow B$ be a function such that A is an infinite set and we wish to check the injectivity of f . In such a case it is not possible to list the images of all elements of set A to see whether different elements of A have different images or not. The following algorithm provides a systematic procedure to check the injectivity of a function.

Algorithm

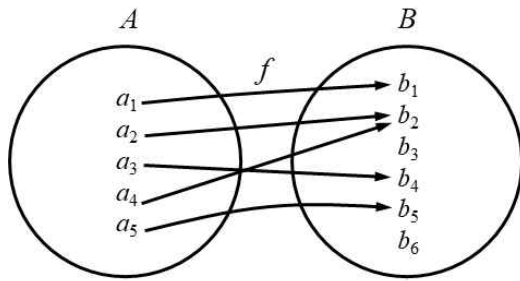
- (i) Take two arbitrary elements x, y (say) in the domain of f .
- (ii) Put $f(x) = f(y)$
- (iii) Solve $f(x) = f(y)$. If $f(x) = f(y)$ gives $x = y$ only, then $f : A \rightarrow B$ is a one-one function (or an injection). Otherwise not.

MANY-ONE FUNCTION

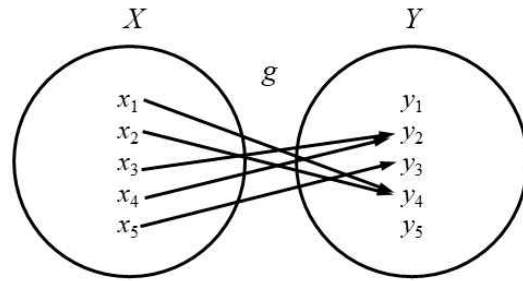
A function $f : A \rightarrow B$ is said to be a many-one function if two or more elements of set A have the same image in B .

Thus, $f : A \rightarrow B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but $f(x) = f(y)$.

Example : Let $f : A \rightarrow B$ and $g : X \rightarrow Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, $a_2 \neq a_4$ but $f(a_2) = f(a_4)$ and $x_1 \neq x_2$ but $g(x_1) = g(x_2)$.
So, f and g are many-one functions.

NOTE In other words, $f : A \rightarrow B$ is many-one function if it is not a one-one function.

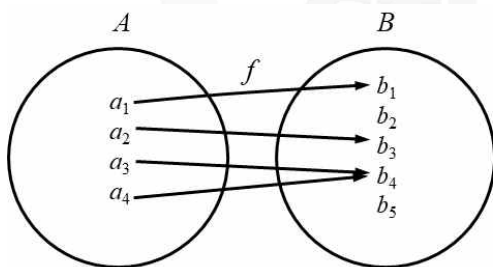
ONTO FUNCTION (SURJECTION)

A function $f : A \rightarrow B$ is said to be an onto function or a surjection if every element of B is the f -image of some element of A i.e., if $f(A) = B$ or range of f is the co-domain of f .
Thus, $f : A \rightarrow B$ is a surjection iff for each $b \in B$, there exists $a \in A$ such that $f(a) = b$.

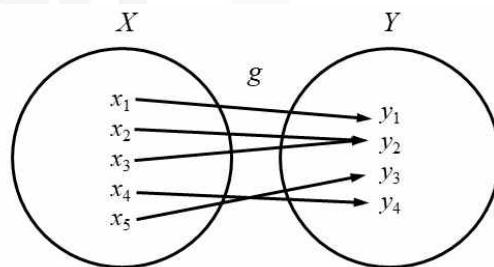
INTO FUNCTION. A function $f : A \rightarrow B$ is an into function if there exists an element in B having no pre-image in A .

In other words, $f : A \rightarrow B$ is an into function if it is not an onto function.

Example : Let $f : A \rightarrow B$ and $g : X \rightarrow Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, b_2 and b_5 are two elements in B which do not have their pre-images in A .

So, $f : A \rightarrow B$ is an into function.

Under function g every elements in Y has its pre-image X . So, $g : X \rightarrow Y$ is an onto function.

The following algorithm can be used to check the surjectivity of a real function.

Algorithm

Let $f : A \rightarrow B$ be the given function.

- (i) Choose an arbitrary element y in B .
- (ii) Put $f(x) = y$
- (iii) Solve the equation $f(x) = y$ for x and obtain x in terms of y . Let $x = g(y)$.
- (iv) If for all values of $y \in B$, the values of x obtained from $x = g(y)$ are in A , then f is onto.
If there are some $y \in B$ for which x , given by $x = g(y)$, is not in A . Then, f is not onto.

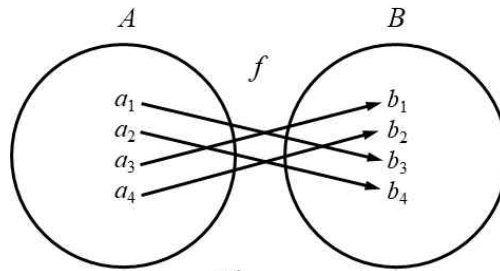
BIJECTION (ONE-ONE ONTO FUNCTION)

A function $f : A \rightarrow B$ is a bijection if it is one-one as well as onto. In other words, a function $f : A \rightarrow B$ is a bijection, if

- (i) it is one-one i.e. $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.
- (ii) it is onto i.e. for all $y \in B$, there exists $x \in A$ such that $f(x) = y$.

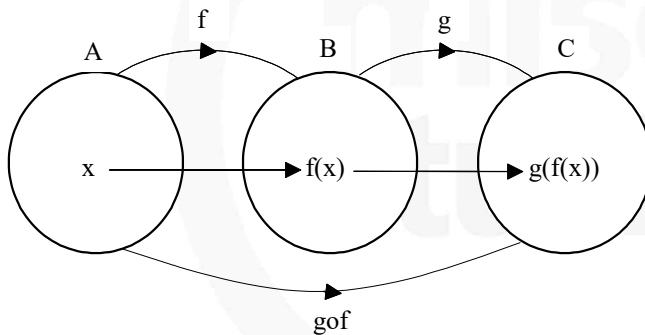
Example : Let $f : A \rightarrow B$ be a function represented by the following diagram :

Clearly, f is a bijection since it is both injective as well as surjective.



Figure

03. Composition of Functions



Definition:

Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. Then a function $gof : A \rightarrow C$ defined by

$$(gof)(x) = g(f(x)), \text{ for all } x \in A$$

is called the composition of f and g .

- NOTE**
- (i) It is evident from the definition that gof is defined only if for each $x \in A$, $f(x)$ is an element of B so that we can take its g -image. Hence, for the composition gof to exist, the range of f must be subset of the domain of g .
 - (ii) It should be noted that gof exists iff the range of f is a subset of domain of g . Similarly, fog exists if range of g is a subset of domain of f .

PROPERTIES OF COMPOSITION OF FUNCTIONS

RESULT 1 The composition of functions is not commutative i.e. $fog \neq gof$.

RESULT 2 The composition of functions is associative i.e. if f, g, h are three functions such that $(fog)oh$ and $fo(goh)$ exist, then

$$(fog)oh = fo(goh)$$

Proof Let A, B, C, D be four non-void sets. Let $h : A \rightarrow B, g : B \rightarrow C$ and $f : C \rightarrow D$ be three functions. Then,

$$\begin{aligned} & h : A \rightarrow B, g : B \rightarrow C, f : C \rightarrow D \\ \Rightarrow & fog : B \rightarrow D \text{ and } h : A \rightarrow B \end{aligned}$$

$$\begin{aligned} \text{Again, } & h, A \rightarrow B, g : B \rightarrow C, f : C \rightarrow D \\ \Rightarrow & fog : C \rightarrow D \text{ and } goh : A \rightarrow C \\ \Rightarrow & fo(goh) : A \rightarrow D \end{aligned}$$

Thus, $(fog)oh$ and $fo(goh)$ are functions from set A to set D .

Now, we shall show that $\{(fog)oh\}(x) = \{fo(goh)\}(x)$ for all $x \in A$.

Let x be an arbitrary element of A and let $y \in B, z \in C$ such that $h(x) = y$ and $g(y) = z$. Then,

$$\begin{aligned} & h : A \rightarrow B, g : B \rightarrow C, f : C \rightarrow D \\ \Rightarrow & fog : B \rightarrow D \text{ and } h : A \rightarrow B \end{aligned}$$

$$\begin{aligned} \text{Again, } & h, A \rightarrow B, g : B \rightarrow C, f : C \rightarrow D \\ \Rightarrow & fog : C \rightarrow D \text{ and } goh : A \rightarrow C \\ \Rightarrow & fo(goh) : A \rightarrow D \end{aligned}$$

Thus, $(fog)oh$ and $fo(goh)$ are functions from set A to set D .

Now, we shall show that $\{(fog)oh\}(x) = \{fo(goh)\}(x)$ for all $x \in A$.

Let x be an arbitrary element of A and let $y \in B, z \in C$ such that $h(x) = y$ and $g(y) = z$. Then,

$$\begin{aligned} & \{(fog)oh\}(x) = (fog)\{h(x)\} \\ \Rightarrow & \{(fog)oh\}(x) = (fog)(y) && [\because h(x) = y] \\ \Rightarrow & \{(fog)oh\}(x) = f(g(y)) \\ \Rightarrow & \{(fog)oh\}(x) = f(z) && \dots(i) \quad [\because g(y) = z] \end{aligned}$$

$$\begin{aligned} \text{And, } & \{fo(goh)\}(x) = f\{goh(x)\} \\ \Rightarrow & \{fo(goh)\}(x) = f\{g(h(x))\} \\ \Rightarrow & \{fo(goh)\}(x) = f\{g(y)\} && [\because h(x) = y] \\ \Rightarrow & \{(fog)oh\}(x) = f(z) && \dots(ii) \quad [\because g(y) = z] \end{aligned}$$

From (i) and (ii), we have

$$\{(fog)oh\}(x) = \{fo(goh)\}(x) \text{ for all } x \in A.$$

Hence, $(fog)oh = fo(goh)$

RESULT 3 The composition of two bijections is a bijection i.e. if f and g are two bijections, then gof is also a bijection.

Proof Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two bijections. Then, gof exists such that $gof : A \rightarrow C$.

We have to prove that gof is injective as well as surjective map.

Injectivity : Let x, y be two arbitrary elements of A . Then,

$$\begin{aligned} \Rightarrow (gof)(x) &= (gof)(y) \\ \Rightarrow g(f(x)) &= g(f(y)) \\ \Rightarrow f(x) &= f(y) && [\because g \text{ is an injective map}] \\ \Rightarrow x &= y && [\because f \text{ is an injective map}] \end{aligned}$$

Thus, $(gof)(x) = (gof)(y)$ for all, $x, y \in A$

So, gof is an injective map.

Surjectivity : In order to prove the surjectivity of gof , we have to show that every element in C has its pre-image in A i.e. for all $z \in C$, there exists $x \in A$ such that $(gof)(x) = z$.

Let z be an arbitrary element of C . Then,

$$\begin{aligned} z \in C &\Rightarrow \text{there exists } y \in B \text{ s.t. } g(y) = z && [\because g \text{ is a surjective map}] \\ \text{and, } y \in B &\Rightarrow \text{there exists } x \in A \text{ s.t. } f(x) = y && [\because f \text{ is a surjective map}] \end{aligned}$$

Thus, we find that for every $z \in C$, there exists $x \in A$ such that

$$(gof)(x) = g(f(x)) = g(y) = z.$$

i.e. every element of C is the gof -image of some element of A .

So, gof is a surjective map.

RESULT 4 Let $f : A \rightarrow B$. Then, $f \circ I_A = I_B \circ f$ i.e. the composition of any function with the identity function is the function itself.

Proof Since $I_A : A \rightarrow A$ and $f : A \rightarrow B$, therefore, $f \circ I_A : A \rightarrow B$. Now let x be an arbitrary element of A . Then,

$$\begin{aligned} (f \circ I_A)(x) &= f(I_A(x)) = f(x) && [\because I_A(x) = x \text{ for all } x \in A] \\ \therefore f \circ I_A &= f \end{aligned}$$

Again, $f : A \rightarrow B$ and $I_B : B \rightarrow B$ of $I_B \circ f : A \rightarrow B$.

Now, let x be an arbitrary element of B . Let $f(x) = y$. Then, $y \in B$

$$\begin{aligned} \therefore (I_B \circ f)(x) &= I_B(f(x)) \\ \Rightarrow (I_B \circ f)(x) &= I_B(y) && [\because f(x) = y] \\ \Rightarrow (I_B \circ f)(x) &= y \\ \Rightarrow (I_B \circ f)(x) &= f(x) && [\because I_B(y) = y \text{ for all } y \in B] \\ \therefore I_B \circ f &= f \end{aligned}$$

Hence, $f \circ I_A = I_B \circ f = f$

RESULT 5 Let $f : A \rightarrow B$, $g : B \rightarrow A$ be two functions such that $gof = I_A$. Then, f is an injection and g is a surjection.

Proof f is an injection : Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$$\begin{aligned} f(x) &= f(y) \\ \Rightarrow g(f(x)) &= g(f(y)) \\ \Rightarrow gof(x) &= gof(y) \\ \Rightarrow I_A(x) &= I_A(y) && [\because gof = I_A \text{ (Given)}] \\ \Rightarrow x &= y && [\text{By def. of } I_A] \end{aligned}$$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$

So, f is an injective map.

g is a surjection : We have, $g : B \rightarrow A$. In order to prove that g is a surjection. It is sufficient to prove that every element in A has its pre-image in B .

Let x be an arbitrary element of A . Then, as $f : A \rightarrow B$ is a function therefore $f(x) \in B$.

Let $f(x) = y$. Then,

$$\begin{aligned} g(y) &= g(f(x)) \\ \Rightarrow g(y) &= gof(x) \\ \Rightarrow g(y) &= I_A(x) && [\because gof = I_A] \\ \Rightarrow g(y) &= x \end{aligned}$$

Thus, for every $x \in A$ there exists $y = f(x) \in B$ such that $g(y) = x$.

So, g is a surjection.

RESULT 6 Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be two function such that $fog = I_B$. Then, f is a surjection and g is an injection.

Proof f is a surjection : In order to prove that $f : A \rightarrow B$ is a surjection, it is sufficient to prove that every element in B has its pre-image in A .

Let b be an arbitrary element of B .

Since $g : B \rightarrow A$. Therefore, $g(b) \in A$.

$$\begin{aligned} \text{Let } g(b) &= a. \\ \text{Now, } f(a) &= f(g(b)) && [\because a = g(b)] \\ \Rightarrow f(a) &= fog(b) \\ \Rightarrow f(a) &= I_B(b) && [\because fog = I_B] \\ \Rightarrow f(a) &= b \end{aligned}$$

Thus, for every $b \in B$ there exists $a \in A$ such that $f(a) = b$.

So, f is a surjection.

g is an injection : Let x, y be any two elements of B such that $g(x) = g(y)$. Then,

$$\begin{aligned} & g(x) = g(y) \\ \Rightarrow & f(g(x)) = f(g(y)) \\ \Rightarrow & f \circ g(x) = f \circ g(y) \\ \Rightarrow & I_B(x) = I_B(y) \\ \Rightarrow & x = y \end{aligned}$$

Thus, $g(x) = g(y) \Rightarrow x = y$ for all $x, y \in B$.

So, g is an injection.

RESULT 7 Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. Then,

(i) $g \circ f : A \rightarrow C$ is onto $\Rightarrow g : B \rightarrow C$ is onto

Proof In order to prove that $g : B \rightarrow C$ is onto whenever $g \circ f : A \rightarrow C$ is onto, it is sufficient to prove that for all $z \in C$ there exists $y \in B$ such that $g(y) = z$.

Let z be an arbitrary element of C . Since $g \circ f : A \rightarrow C$ is onto. Therefore, there exists $x \in A$ such that

$$\begin{aligned} & g \circ f(x) = z \\ \Rightarrow & g(f(x)) = z \\ \Rightarrow & g(y) = z, \text{ where } y = f(x) \in B. \end{aligned}$$

Thus, for all $z \in C$, there exists $y = f(x) \in B$ such that $g(y) = z$.

Hence, $g : B \rightarrow C$ is onto.

(ii) $g \circ f : A \rightarrow C$ is one-one $\Rightarrow f : A \rightarrow B$ is one-one.

Proof In order to prove that $f : A \rightarrow B$ is one-one, it is sufficient to prove that $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$$\begin{aligned} & f(x) = f(y) \\ \Rightarrow & g(f(x)) = g(f(y)) && [\because g : B \rightarrow C \text{ is a function}] \\ \Rightarrow & g \circ f(x) = g \circ f(y) \\ \Rightarrow & x = y && [\because g \circ f : A \rightarrow C \text{ is a function}] \end{aligned}$$

Hence, $f : A \rightarrow B$ is one-one.

(iii) $g \circ f : A \rightarrow C$ is onto and $\Rightarrow g : B \rightarrow C$ is one-one $\Rightarrow f : A \rightarrow B$ is onto.

Proof In order to prove that $f : A \rightarrow B$ is onto, it is sufficient to prove that for all $y \in B$ there exists $x \in A$ such that $f(x) = y$.

Let y be an arbitrary element of B . Then,

$$g(y) \in C \quad [\because g : B \rightarrow C]$$

Since $g \circ f : A \rightarrow C$ is an onto function. Therefore, for any $g(y) \in C$ there exists $x \in A$ such that $g \circ f(x) = g(y)$

$$\Rightarrow g(f(x)) = g(y)$$

$$\Rightarrow f(x) = y \quad [\because g \text{ is one-one}]$$

Thus, for all $y \in B$ there exists $x \in A$ such that $f(x) = y$.

Hence, $f : A \rightarrow B$ is onto.

(iv) $g \circ f : A \rightarrow C$ is one-one and $\Rightarrow f : A \rightarrow B$ is onto $\Rightarrow g : B \rightarrow C$ is one-one.

Proof Let $y_1, y_2 \in B$ such that $g(y_1) = g(y_2)$. In order to prove that g is one-one, it is sufficient to prove that $y_1 = y_2$.

Now, $f : A \rightarrow B$ is onto and $y_1, y_2 \in B$. So, there exist $x_1, x_2 \in A$ such that

$$f(x_1) = y_1 \text{ and } f(x_2) = y_2$$

$$\text{Now, } g(y_1) = g(y_2)$$

$$\Rightarrow g(f(x_1)) = g(f(x_2))$$

$$\Rightarrow g \circ f(x_1) = g \circ f(x_2)$$

$$\Rightarrow x_1 = x_2 \quad [\because g \circ f : A \rightarrow C \text{ is one-one}]$$

$$\Rightarrow f(x_1) = f(x_2) \quad [\because f : A \rightarrow B \text{ is a function}]$$

$$\Rightarrow y_1 = y_2$$

Hence, $g : B \rightarrow C$ is one-one.

04. Inverse of An Element

Let A and B be two sets and let $f : A \rightarrow B$ be a mapping. If $a \in A$ is associated to $b \in B$ under the function f , then ' b ' is called the f image of ' a ' and we write it as $b = f(a)$. We also say that ' a ' is the pre-image or inverse element of ' b ' under f and we write $a = f^{-1}(b)$.

NOTE The inverse of an element under a function may consist of a single element, two or more elements or no element depending on whether function is injective or many-one; onto or into.

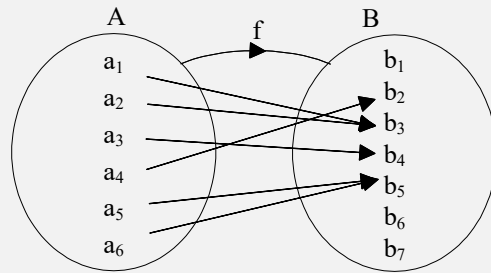
If f is represented by Figure, then we find that

$$f^{-1}(b_1) = \phi, f^{-1}(b_2) = a_4,$$

$$f^{-1}(b_3) = \{a_1, a_2\}, f^{-1}(b_4) = a_3,$$

$$f^{-1}(b_5) = \{a_5, a_6\}, f^{-1}(b_6) = \phi$$

and, $f^{-1}(b_7) = \phi$



05. Inverse of A Function

Definition:

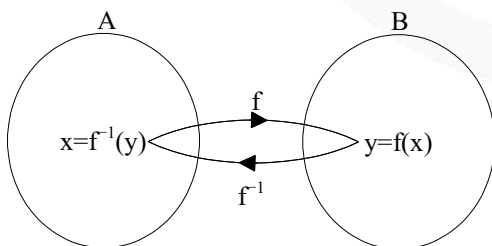
Let $f: A \rightarrow B$ be a bijection. Then a function $g: B \rightarrow A$ which associates each element $y \in B$ to a unique element $x \in A$ such that $f(x) = y$ is called the inverse of f .

i.e., $f(x) = y \Leftrightarrow g(y) = x$

The inverse of f is generally denoted by f^{-1}

Thus, if $f: A \rightarrow B$ is a bijection, then $f^{-1}: B \rightarrow A$ is such that

$$f(x) = y \Leftrightarrow f^{-1}(y) = x$$



In order to find the inverse of a bijection, we may follow the following algorithm.

Algorithm

Let $f: A \rightarrow B$ be a bijection. To find the inverse of f we follow the following steps:

STEP I Put $f(x) = y$, where $y \in B$ and $x \in A$.

STEP II Solve $f(x) = y$ to obtain x in terms of y .

STEP III In the relation obtained in step II replace x by $f^{-1}(y)$ to obtain the required inverse of f .

06. Properties of Inverse of a Function

RESULT 1 If $f: A \rightarrow B$ is a bijection and $g: B \rightarrow A$ is the inverse of f , then $f \circ g = I_B$ and $g \circ f = I_A$, where I_A and I_B are the identity function on the sets A and B respectively.

Proof In order to prove that $g \circ f = I_A$ and $f \circ g = I_B$, we have to prove that $(g \circ f)(x) = x$ for all $x \in A$ and $(f \circ g)(y) = y$ for all $y \in B$.

Let x be an element of A such that $f(x) = y$. Then,

$$g(y) = x \quad [\because g \text{ is inverse of } f]$$

Now, $(g \circ f)(x) = g(f(x)) = g(y) = x$

$$(g \circ f)(x) = x \text{ for all } x \in A$$

$$\Rightarrow g \circ f = I_A.$$

We have,

$$(f \circ g)(y) = f(g(y)) = f(x) = y$$

$$\therefore f \circ g(y) = y \text{ for all } y \in B$$

$$\Rightarrow f \circ g = I_B.$$

Hence, $g \circ f = I_A$ and $f \circ g = I_B$.

RESULT 2 If $f: A \rightarrow B$ and $g: B \rightarrow C$ are two bijections, then $g \circ f: A \rightarrow C$ is a bijection and

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

Proof We have,

$$\left. \begin{array}{l} f: A \rightarrow B \text{ is a bijection} \\ g: B \rightarrow C \text{ is a bijection} \end{array} \right\} \Rightarrow g \circ f: A \rightarrow C \text{ is a bijection} \Rightarrow (g \circ f)^{-1}: C \rightarrow A \text{ exists.}$$

Again,

$$\left. \begin{array}{l} f: A \rightarrow B \text{ is a bijection} \Rightarrow f^{-1}: B \rightarrow A \text{ is a bijection} \\ g: B \rightarrow C \text{ is a bijection} \Rightarrow g^{-1}: C \rightarrow B \text{ is a bijection} \end{array} \right\} \Rightarrow f^{-1} \circ g^{-1}: C \rightarrow A$$

Let $x \in A$, $y \in B$ and $z \in C$ such that $f(x) = y$ and $g(y) = z$. Then,

$$(g \circ f)(x) = g(f(x)) = g(y) = z$$

$$\Rightarrow (g \circ f)^{-1}(z) = x \quad \dots(i)$$

Now,

$$\begin{aligned} f(x) &= y \text{ and } g(y) = z \\ \Rightarrow f^{-1}(y) &= x \text{ and } g^{-1}(z) = y \\ \therefore (f^{-1} \circ g^{-1})(z) &= (f^{-1}(g^{-1}(z))) = f^{-1}(y) = x \end{aligned} \quad \dots(\text{ii})$$

From (i) and (ii), we have

$$(g \circ f)^{-1}(z) = (f^{-1} \circ g^{-1})(z) \text{ for all } z \in C.$$

$$\text{Hence, } (g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

RESULT 3 If $f: A \rightarrow B$ and $g: B \rightarrow A$ be two functions such that $g \circ f = I_A$ and $f \circ g = I_B$.

Then, f and g are bijections and $g = f^{-1}$.

Proof f is one-one : Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$$\begin{aligned} f(x) &= f(y) \\ \Rightarrow g(f(x)) &= g(f(y)) \\ \Rightarrow (g \circ f)(x) &= (g \circ f)(y) \\ \Rightarrow I_A(x) &= I_A(y) \\ \Rightarrow x &= y \\ \therefore f &\text{ is a one-one map.} \end{aligned}$$

f is onto : Let $y \in B$ and let $g(y) = x$. Then,

$$\begin{aligned} g(y) &= x \\ \Rightarrow f(g(y)) &= f(x) \\ \Rightarrow (f \circ g)(y) &= f(x) \\ \Rightarrow I_B(y) &= f(x) && [\because f \circ g = I_B] \\ \Rightarrow y &= f(x) && [\because I_B(y) = y] \end{aligned}$$

Thus, for each $y \in B$, there exists $x \in A$ such that $f(x) = y$.

So, f is onto.

Hence, f is a bijection.

Similarly, it can be proved that g is a bijection.

Now we shall show that $g = f^{-1}$.

Since $f: A \rightarrow B$ is a bijection. Therefore, f^{-1} exists.

$$\begin{aligned} \text{Now, } f \circ g &= I_B \\ \Rightarrow f^{-1} \circ (f \circ g) &= f^{-1} \circ I_B \\ \Rightarrow (f^{-1} \circ f) \circ g &= f^{-1} \circ I_B && [\text{By associativity}] \\ \Rightarrow I_A \circ g &= f^{-1} \circ I_B && [\because f^{-1} \circ f = I_A] \\ \Rightarrow g &= f^{-1} && [\because I_A \circ g = g \text{ and } f^{-1} \circ I_B = f^{-1}] \end{aligned}$$

Hence, $g = f^{-1}$

07. Binary Operation

DEFINITION

A binary operation $*$ on a set A is a function $*$: $A \times A \rightarrow A$. We denote $*$ (a, b) by $a * b$.

Eg. : Addition, subtraction and multiplication are binary operations on \mathbf{R} , but division is not a binary operation on \mathbf{R} . Further division is a binary operation on the set \mathbf{R} of non-zero real numbers.

Solution $+$: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ is given by

$$(a, b) \rightarrow a + b$$

$-$: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ is given by

$$(a, b) \rightarrow a - b$$

\times : $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ is given by

$$(a, b) \rightarrow ab$$

Since '+', '-' and '×' are functions, they are binary operation on \mathbf{R} .

But \div : $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$, given by $(a, b) \rightarrow \frac{a}{b}$ is a function and hence a binary operation on \mathbf{R} .

Types of Binary Operation

Type I Commutativity A binary operation '*' on a set S is said to be a commutative binary operation, if

$$a * b = b * a \text{ for all } a, b \in S$$

Eg.: The binary operations addition (+) and multiplication (×) are commutative binary operation on Z . However, the binary operation subtraction (-) is not a commutative binary operation on Z as $3 - 2 \neq 2 - 3$.

Type II Associativity A binary operation '*' on a set S is said to be an associative binary operation, if

$$(a * b) * c = a * (b * c) \text{ for all } a, b \in S.$$

Eg.: The binary operations of addition (+) and multiplication (×) are associative binary operation on Z . However, the binary operation subtraction (-) is not a associative binary operation on Z as $(2 - 3) - 5 \neq 2 - (3 - 5)$.

Identity Element

Let "*" be a binary operation on a set S . If there exists an elements $e \in S$ such that $a * e = a = e * a$ for all $a \in S$.

Then, e is called an identity element for the binary operation "*" on set S .

Eg. : Consider the binary operation of addition (+) on Z . we know that $0 \in Z$ such that

$$a + 0 = a = 0 + a \text{ for all } a \in Z$$

So, '0' is the identity element for addition on Z .

If we consider multiplication on Z , then '1' is the identity element for multiplication on Z , because

$$1 \times a = a = a \times 1 \text{ for all } a \in Z.$$

Eg. : We know that addition (+) and multiplication (\cdot) are binary operations on N such that

$$n \times 1 = n = 1 \times n \text{ for all } n \in N$$

But, there do not exist any natural number e such that

$$n + e = n = e + n \text{ for all } n \in N.$$

So, 1 is the identity element for multiplication on N , but N does not have identity element for addition on N .

Inverse of an Element

Invertible Element Let '*' be a binary operation on a set S , and let e be the identity element in S for this binary operation * on S . Then, an element $a \in S$ is called an invertible element if there exists an element $b \in S$ such that

$$a * b = e = b * a$$

The element b is called an inverse of element a .

Thus, an element $b \in S$ is called an inverse of an element $a \in S$, if

$$a * b = e = b * a.$$

Eg.: Consider the binary operation addition (+) on Z . Clearly, 0 is the identity element for addition on Z and for any integer a , we have

$$a + (-a) = 0 = (-a) + a$$

So, $-a$ is the inverse of $a \in Z$.

Multiplication is also a binary operation on Z and 1 is the identity element for multiplication on Z . But, no element, other than $1 \in Z$, is invertible.

Composition Table

A binary operation on finite set can be completely describe by means of a table known as a composition table. Let $S = \{a_1, a_2, \dots, a_n\}$ be a finite set and * be a binary operation on S . Then the composition table for * is constructed in the manner indicated below.

We write the elements a_1, a_2, \dots, a_n of the set S in the top horizontal row and the left vertical column in the same order. Then we put down the element $a_i * a_j$ at the intersection of the row headed by a_i ($1 \leq j \leq n$) to get the following table :

*	a_1	a_2	...	a_i	...	a_j	...	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$...	$a_1 * a_i$...	$a_1 * a_j$...	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$...	$a_2 * a_i$...	$a_2 * a_j$...	$a_2 * a_n$
⋮								
a_i	$a_i * a_1$	$a_i * a_2$...	$a_i * a_i$...	$a_i * a_j$...	$a_i * a_n$
⋮								
a_j	$a_j * a_1$	$a_j * a_2$...	$a_j * a_i$...	$a_j * a_j$...	$a_j * a_n$
⋮								
a_n	$a_n * a_1$	$a_n * a_2$...	$a_n * a_i$...	$a_n * a_j$...	$a_n * a_n$

- (i) From the composition table we infer the following results.

If all the entries of the table are elements of set S and each element of S appears once and only once in each row and in each column, then the operation is a binary operation.

Sometimes we also say that the binary operation is well defined which means that the operation*. But for us, this is a consequence of the definition of binary operation.

- (ii) If the entries in the table are symmetric with respect to the diagonal which starts at the upper left corner of the table and terminates at the lower right corner, we say that the binary operation is commutative on S , otherwise it is said to be non-commutative on S .
- (iii) If the row headed by an element say a_j coincides with the row at the top and the column headed by a_j coincides with the column on extreme left, then a_j is the identity element in S for the binary operation * on S .
- (iv) If each row except the top most row or each column except the left most column contains the identity element every element of S is invertible with respect to *. To find the inverse of an element say a_i , we consider row (or column) headed by a_i . Then we determine the position of identity element e in this row (or column). If e appears in the column (or row) headed by a_j , then a_i and a_j are inverse of each other.

CBSE Exam Pattern Exercise Subjective Questions (1)

(Q 1 to 4) One Mark

- Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.
- If $f : R \rightarrow R$ is defined by $f(x) = (3 - x^3)^{1/3}$, then find $f \circ f(x)$.
- Let $*$: $R \times R \rightarrow R$ given by $(a, b) \rightarrow a + 4b^2$ be a binary operation. Compute $(-5) * (2 * 0)$.
- If $*$ is a binary operation on set Q of rational number defined as $a * b = \frac{ab}{5}$. Write the identity for $*$, if any.

(Q 5 to 8) Four Marks

- If the function $f : R \rightarrow R$ is given by $f(x) = x^2 + 2$ and $g : R \rightarrow R$ is given by $g(x) = \frac{x}{x-1}$; $x \neq 1$, then find $f \circ g$ and $g \circ f$, and hence find $f \circ g(2)$ and $g \circ f(-3)$.
- Show that $f : N \rightarrow N$, given by $f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$ is bijective (both one-one and onto).
- Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.
- Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min \{a, b\}$. Write operation table of operation $*$.

(Q 9 to 10) Six Marks

- Let $f : N \rightarrow R$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \rightarrow S$, where S is the range of f , is invertible. Also, find the inverse of f .
- Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to

* on A

- (i) Find the identity element in A .
- (ii) Find the invertible elements of A .





Answer & Solution

Q1.

$$\begin{aligned}
 [0] &= \{b \in A; b \in \mathbb{R}\} \\
 &= \{b \in A; (b, 0) \in R\} \\
 &= \{b \in A; 2 \text{ divides } b - 0\} \\
 &= \{0, 2, 4\}
 \end{aligned}$$

Q2.

$$\begin{aligned}
 f \circ f(x) &= f(3 - x)^{1/3} \\
 &= (3 - y^3)^{1/3} \text{ where } y = (3 - x^3)^{1/3} \\
 &= (3 - ((3 - x^3)^{1/3})^3)^{1/3} \\
 &= (3 - (3 - x^3))^{1/3} = (x^3)^{1/3} = x = I(x) \\
 \therefore f \circ f &= I_R
 \end{aligned}$$

Q3.

$$\begin{aligned}
 a * b &= a + 4b^2 \\
 &\Rightarrow -5 * (2 * 0) \\
 &\Rightarrow -5 * (2 + 4(0^2)) \\
 &= -5 * 2 \\
 &= -5 + 4(2)^2 \\
 &= -5 + 16 = 11
 \end{aligned}$$

Q4.

Let e be the identity element in Q under $*$

$$\begin{aligned}
 \Rightarrow a * e &= a \quad \forall a \in Q \\
 &= e * a
 \end{aligned}$$

$$\Rightarrow \frac{ae}{5} = a \quad \forall a \in Q$$

$$\Rightarrow e = 5 \in Q$$

 \therefore Identity element = 5

Q5.

$$f(x) = x^2 + 2 \quad \forall x \in \mathbb{R}$$

$$D(f) = \mathbb{R}$$

$$R(f) = [2, \infty]$$

03 Relations & Function

$$(\because x^2 \geq 0 \forall x \in \mathbf{R} \Rightarrow x^2 + 2 \geq 2 \forall x \in \mathbf{R} \therefore x^2 + 2 \in [2, \infty))$$

$$g(x) = \frac{x}{x-1} \text{ \& } g(x) \notin \mathbf{R} \text{ for } x = 1$$

$$\therefore D(g) = \mathbf{R}/\{1\}$$

$$\text{\& } R(g) = \mathbf{R}/\{1\}.$$

$$\left(\begin{array}{l} \frac{x}{x-1} = y \Rightarrow x = yx - y \\ \Rightarrow x = \frac{y}{y-1} \in R \forall y \in R/\{1\} \end{array} \right)$$

$$\text{As } R(g) \subseteq D(f)$$

$$\therefore fog \text{ exists}$$

$$\text{for } x \in D(g) = \mathbf{R} - \{1\},$$

$$(fog)(x) = f(g(x))$$

$$= f\left(\frac{x}{x-1}\right)$$

$$= \left(\frac{x}{x-1}\right)^2 + 2$$

$$= \frac{x^2}{(x-1)^2} + 2$$

$$= \frac{3x^2 - 4x + 2}{(x-1)^2}$$

$$\therefore fog(2) = \frac{3(2)^2 - 4(2) + 2}{(2-1)^2} = 6$$

$$\text{And, as } R(g) \subseteq D(f)$$

$$\therefore gof \text{ is defined}$$

$$\therefore \text{for } x \in \mathbf{R} = D(f)$$

$$gof(x) = g(f(x))$$

$$= g(x^2 + 2) = \frac{x^2 + 2}{x^2 + 2 - 1}$$

$$= \frac{x^2 + 2}{x^2 + 1}$$

$$\therefore gof(-3) = \frac{(-3)^2 + 2}{(-3)^2 + 1} = \frac{11}{10}$$

Q6.

Injective :

case (i) if x_1 & x_2 both are odd.

$$\text{Then, } f(x_1) = f(x_2)$$

$$\Rightarrow x_1 + 1 = x_2 + 1$$

$$\Rightarrow x_1 = x_2$$

case (ii) if x_1 & x_2 both are even

$$\text{Then, } f(x_1) = f(x_2)$$

$$\Rightarrow x_1 - 1 = x_2 - 1$$

$$\Rightarrow x_1 = x_2$$

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case (iii) If x_1 is even & x_2 is odd

$$\Rightarrow x_1 \neq x_2$$

to show $f(x_1) \neq f(x_2)$

$$\begin{aligned} \text{As } x_1 \neq x_2 &\Rightarrow x_1 - 1 \neq x_2 + 1 \\ &\Rightarrow f(x_1) \neq f(x_2) \end{aligned}$$

\therefore From all three cases f is one-one

Surjective :

$$\text{let } y \in \mathbf{N} = C(f)$$

If y is odd then $y + 1$ is an even natural no ;

$$f(y + 1) = (y + 1) - 1 = y$$

$\therefore \exists x = y + 1 \in \mathbf{N}$ such that

$$f(x) = y \in \mathbf{N}$$

And, if y is even then $y - 1$ is an odd natural no;

$$f(y - 1) = (y - 1) + 1 = y$$

$\therefore \exists x = y - 1 \in \mathbf{N}; f(x) = y \in \mathbf{N}$

$\therefore f$ is onto.

Q7.

Reflexive :

Claim : $1/2 \bar{R} 1/2$

$$\because \frac{1}{2} \not\leq \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\therefore 1/2 \bar{R} 1/2$

$\therefore (1/2, 1/2) \notin R$

$\therefore R$ is not reflexive.

Symmetric :

Claim : $-2 \bar{R} 3$ but $3 \bar{R} -2$

As $-2 \leq 3^3 \Rightarrow (-2, 3) \in R$

But $3 \not\leq (-2)^3 = -8 \therefore$

$(3, -2) \notin R$

$\therefore R$ is not symmetric.

Transitive :

Claim : $2 \bar{R} 3/2$ & $3/2 \bar{R} 4/3$ but $2 \bar{R} 4/3$

As $2 \leq (3/2)^3$ & $3/2 \leq (4/3)^3$ but $2 \not\leq (4/3)^3$

$\Rightarrow (2, 3/2) \in R, (3/2, 4/3) \in R$ but $(2, 4/3) \notin R$

$\therefore R$ is not transitive.

Q8.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Q9.

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ONE ONE :

$$\begin{aligned} \text{Let } f(x_1) &= f(x_2) \\ \Rightarrow 9x_1^2 + 6x_1 - 5 &= 9x_2^2 + 6x_2 - 5 \\ \Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 &= 0 \\ 9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) &= 0 \\ (x_1 - x_2)(3x_1 + 3x_2 + 2) &= 0 \\ \text{But } 3x_1 + 3x_2 + 2 &\neq 0 \\ \therefore x_1, x_2 &\in \mathbf{N} \\ \Rightarrow x_1 &= x_2 \\ \therefore f &\text{ is one one} \end{aligned}$$

ONTO :

$$\begin{aligned} \text{As } S &= R(f) \\ &\& f : \mathbf{N} \rightarrow S \\ \therefore C(f) &= R(f) \\ \therefore f &\text{ is onto} \\ \therefore f &\text{ is invertible } \Rightarrow \exists f^{-1} : S \rightarrow \mathbf{N} \end{aligned}$$

$$\begin{aligned} \text{Let } y &= 9x^2 + 6x - 5 \\ y &= (3x)^2 + 2 \cdot 3x \cdot 1 + (1)^2 - (1)^2 - 5 \\ y &= (3x + 1)^2 - 6 \\ \Rightarrow y + 6 &= (3x + 1)^2 \\ \Rightarrow 3x + 1 &= +\sqrt{y+6} \\ (\because x &\in \mathbf{N}) \\ \therefore 3x &= \sqrt{y+6} - 1 \\ x &= \frac{\sqrt{y+6} - 1}{3} \\ \therefore f^{-1}(x) &= \frac{\sqrt{x+6} - 1}{3} \end{aligned}$$

Q10.

Commutative :

$$\begin{aligned} \text{let } (a, b), (c, d) &\in Q \times Q = A \\ \Rightarrow (a, b) * (c, d) &= (ac, b + ad) \\ \&\ (c, d) * (a, b) &= (ca, d + bc) \\ \text{As } (a, b) * (c, d) &\neq (c, d) * (a, b) \\ \therefore * &\text{ is not commutative} \end{aligned}$$

Associative :

$$\begin{aligned} \text{Let } (a, b), (c, d), (e, f) &\in Q \times Q \\ \text{Consider} \end{aligned}$$

$$\begin{aligned} ((a, b) * (c, d)) * (e, f) &= (ac, b + ad) * (e, f) = (ace, b + ad + acf) = (a, b) * ((c, d) * (e, f)) \\ \therefore * &\text{ is associative.} \end{aligned}$$

Identity Element

$$\begin{aligned} \text{Let } (e, f) &\text{ be the identity element in } A \text{ w.r.t. } * \\ \Rightarrow (a, b) * (e, f) &= (a, b) \quad \forall (a, b) \in A = Q \times Q \\ \Rightarrow (ae, b + af) &= (a, b) \\ \Rightarrow ae = a \&\ b + af &= b \end{aligned}$$

$$\Rightarrow e = 1 \text{ \& } f = 0$$

$$\therefore (1, 0) \in Q \times Q = A$$

is the identity element of A w.r.t. *

Inverse Element

let $(c, d) \in Q \times Q = A$ be the inverse element of $(a, b) \in Q \times Q$

$$\Rightarrow (a, b) * (c, d) = (1, 0)$$

$$\Rightarrow (ac, b + ad) = (1, 0)$$

$$\Rightarrow ac = 1 \text{ \& } b + ad = 0$$

$$\Rightarrow c = 1/a \text{ \& } d = -b/a ; a \neq 0$$

\therefore for $(a, b) \in Q \times Q$; $a \neq 0$, inverse of (a, b) exists & is given by $(1/a, -b/a) \in Q \times Q$.



CBSE Exam Pattern Exercise Objective Questions (2)

- Which of the following functions are equal
 - $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1}x)$
 - $\frac{x^2-4}{x-2}, x+2$
 - $\frac{x^2}{x}, x$
 - $A=\{1,2\}, B=\{3,6\}$
 $f:A \rightarrow B$ given by $f(x) = x^2+2$ and
 $g:A \rightarrow B$ given by $g(x) = 3x$
- Let $f : \left[\frac{1}{2}, \infty\right) \rightarrow \left[\frac{3}{4}, \infty\right)$, where $f(x) = x^2 - x + 1$ is
 - one-one onto
 - many one-into
 - many one-onto
 - one-one into
- IF $A = \{1,2,3\}$
 $B = \{4,5,6,7\}$ and
 $f = \{(1,4) (2,5) (3,6)\}$ is a function from A to B then f is
 - one-one
 - onto
 - many one
 - both (a) and (b)
- The range of the function $f(x) = \frac{|x-2|}{x-2}, x \neq 2$ is
 - $\{1,0,-1\}$
 - $\{1\}$
 - $\{1,-1\}$
 - None of these
- $f:\mathbb{R} \rightarrow \mathbb{R}$ and $g:\mathbb{R} \rightarrow \mathbb{R}$ are given by $f(x) = |x|$ and $g(x) = |5x-2|$, then $f \circ g$ is
 - $|5x-2|$
 - $5x-2$
 - $2-5x$
 - None of these

6. Range of the function $f(x) = \frac{|x^2+1|}{x^2+1}$ is
- (a) $\{1\}$
 - (b) $\{1,-1\}$
 - (c) $\{1,0,-1\}$
 - (d) \mathbb{R}
7. If set A has 5 elements and set B has three elements then total no. of one-one functions from A to B are
- (a) 0
 - (b) 5P_3
 - (c) 5
 - (d) $5!$
8. If $f(x) = [x]$ and $g(x) = |x|$ then $\text{fog} \left(\frac{-5}{2} \right)$ is
(where $[.]$ represents greatest integer function of x)
- (a) 2
 - (b) 3
 - (c) -3
 - (d) -2



Answer & Solution

1. (d)

(a) $\sin^{-1}(\sin x) \neq \sin(\sin^{-1}x)$

 \because Those functions are equal

whose range & domain are equal

But $\sin^{-1}(\sin x) = x \Rightarrow x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

But $\sin(\sin^{-1}x) = x \Rightarrow x \in [-1, 1]$

(b) $\frac{(x^2 - 4)}{(x - 2)} = x + 2, x \neq 2$

Domain of $\frac{x^2 - 4}{x - 2} \Rightarrow \mathbb{R} - 2$

& Domain of $x + 2$ is \mathbb{R}

Hence they are not equal

(c) same explanation as (b)

(d) $A = \{1, 2\}, B = \{3, 6\}$

$f(1) = 3$

$f(2) = 6$

$g(x) = 3x$

$g(1) = 3$

$g(2) = 6$

Since range & domain in both functions is equal.

Hence functions are equal function

2. (a)

$f(x) = x^2 - x + 1$

$f: \left(\frac{1}{2}, \infty \right) \rightarrow \left(\frac{3}{4}, \infty \right)$

For one-one

$f(x_1) = f(x_2)$

$x_1^2 - x_1 + 1 = x_2^2 - x_2 + 1$

$x_1^2 - x_2^2 - x_1 + x_2 = 0$

$(x_1 - x_2)(x_1 + x_2) - 1(x_1 - x_2) = 0$

$(x_1 - x_2)(x_1 + x_2 - 1) = 0$

either $x_1 = x_2$ or $x_1 + x_2 = 1$

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But $x_1 + x_2 = 1$

only when $x_1 = x_2$

& for no other value

$\therefore x_1 = x_2$

Hence one-one

onto

$$f(x) = x^2 - x + 1$$

$$= x^2 - x + \frac{1}{4} - \frac{1}{4} + 1$$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

[Using completing the square method]

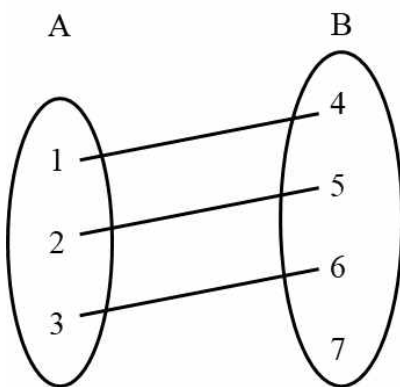
For $x \geq \frac{1}{2}$

$$y \geq \frac{3}{4}$$

Hence range = Codomain

Hence function is onto

3. (a)



Clearly f is one but not onto

4. (c)

$$f(x) = \frac{|x-2|}{x-2}, x \neq 2 \text{ is}$$

$$f(x) = \begin{cases} 1 & x-2 > 0 \\ -1 & x-2 < 0 \end{cases}$$

\therefore Range is $\{1, -1\}$

5. (a)

$$f(x) = |x|$$

$$g(x) = |5x - 2|$$

$$f(g(x)) = ||5x - 2||$$

$$= |5x - 2|$$

6. (a)

$$f(x) = \frac{|x^2 + 1|}{x^2 + 1}$$

$x^2 + 1$ is always +ve

$$\therefore f(x) = 1$$

{1}

7. (a)

Set A has 5 elements

Set B has 3 elements

\therefore one-one function = zero

8. (a)

$$f(x) = [x]$$

$$g(x) = |x|$$

$$f(g(x)) = [|x|]$$

$$f\left(g\left(\frac{-5}{2}\right)\right) = \left[\left| \frac{-5}{2} \right| \right] = \left[\frac{5}{2} \right] = 2$$

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