

CRASH COURSE

CBSE 12th 2021-22

PHYSICS

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(SUBJECTIVE QUESTIONS)

1. How is force between two charges affected when each charge is doubled and distance between them is also doubled?
2. How much positive and negative charge is there in a cup of water?
3. A metal sphere has a charge of $-6.5 \mu\text{C}$. When 5×10^{13} electrons are removed from the sphere, what would be the net charge on it?
4. Define relative permittivity of a medium.
5. Explain the phenomenon of charging by induction?

Answer & Solutions

1.

As $F \propto \frac{|q_1||q_2|}{r^2}$ \therefore F becomes $\frac{(2)(2)}{2^2}$ time = 1 time, i.e., force remains the same.

2.

So in one cup of water (250 ml) there are 1.338×10^7 C positive charge and -1.338×10^7 C negative charge. Totally they will be neutral.

3.

So, 8 uC charge transferred from sphere. so, net charge = $-6.5 + 8 = 1.5$ uC

4.

Relative permittivity $\epsilon(r)$ is defined as the ratio of absolute permittivity of a medium (ϵ) to the absolute permittivity of free space (ϵ_0)

$$\epsilon(r) = \epsilon / \epsilon_0$$

It can also be defined as the ratio of forces of attraction/repulsion between two point charges separated by a certain distance in vacuum to the force of attraction when they are placed in a medium same distance apart.

$$\epsilon(r) = F(o) / F(\text{medium})$$

5.

When charging a conductor by induction, a charged object is brought close to but does not touch the conductor. In the end the conductor has charge of the opposite sign as the charge on the object. One way to carry out the four-step process :

- (i) Bring the charged object close to, but not touching, the conductor. Charge on the conductor shifts in response to the nearby charged object.
- (ii) Connect the conductor to ground. Ground is basically a charge reservoir - anything that can give up or receive charge without noticing the change. Electrons flow from ground to the conductor if the charged object is positive, and the opposite way if the object is negative. The conductor now has a net charge with a sign opposite to the sign on the charged object.
- (iii) Remove the ground connection. The transferred electrons can't get back to where they came from.
- (iv) Remove the charged object. The net charge distributes itself over the surface of the conductor.

(OBJECTIVE QUESTIONS)

- Two equal charges placed in air separated by a distance 3 m repel each other with force 0.1 gf. Calculate the magnitude of either of the charges.
 - $9.9 \times 10^{-7} \text{ C}$
 - $4.5 \times 10^{-7} \text{ C}$
 - $12.4 \times 10^{-7} \text{ C}$
 - $15.2 \times 10^{-7} \text{ C}$
- An electron is released with a velocity of $5 \times 10^6 \text{ ms}^{-1}$ in an electric field of 10^3 NC^{-1} which has been applied so as to oppose its motion. The distance travelled by electron before coming to
 - $7.11 \times 10^{-2} \text{ m}$
 - $3.35 \times 10^{-2} \text{ m}$
 - $9.2 \times 10^{-2} \text{ m}$
 - $11.4 \times 10^{-2} \text{ m}$
- Three charges, each equal to q , are placed at the three corners of a square of side a . Find the electric field at the fourth corner of the square
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{2a^2}$
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{2a^2} (2\sqrt{2} + 1)$
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{3a^2} (\sqrt{2} + 1)$
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{3a^2}$
- If the number of electric lines of force emerging out of a closed surface are 10^3 , calculate the charge enclosed by the surface

(a) $4.42 \times 10^{-9} \text{ C}$	(c) $17.7 \times 10^{-9} \text{ C}$
(b) $8.85 \times 10^{-9} \text{ C}$	(d) $12.5 \times 10^{-9} \text{ C}$
- A uniform electric field $\vec{E} = 5 \times 10^3 \text{ NC}^{-1}$ exist in space. The flux of this field through a square of 10 cm on a side, and whose plane is parallel to the YZ plane is
 - $25 \text{ Nm}^2 \text{ C}^{-1}$
 - $50 \text{ Nm}^2 \text{ C}^{-1}$
 - $75 \text{ Nm}^2 \text{ C}^{-1}$
 - $100 \text{ Nm}^2 \text{ C}^{-1}$

Answer & Solutions

1. (a)

For $q_1 = q_2 = q$ (say),

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \times q}{r^2}$$

$$\text{or } q = \sqrt{4\pi\epsilon_0 F \times r}$$

Setting $F = 0.1 \text{ gf} = 0.1 \times 10^{-3} \text{ kgf} = 0.1 \times 10^{-3} \times 9.8 \text{ N}$,

$$r = 3 \text{ m and } \epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1} \text{ m}^{-1}$$

We get

$$q = 9.9 \times 10^{-7} \text{ C}$$

2. (a)

Since electric field is applied so as to oppose the motion of electron,

$$a = -\frac{eE}{m} = -\frac{1.6 \times 10^{-19} \times 10^3}{9.1 \times 10^{-31}}$$

$$= -1.758 \times 10^{14} \text{ m s}^{-2} \text{ (retardation)}$$

Now, $u = 5 \times 10^6 \text{ m s}^{-1}$, $v = 0$

Using the relation :

$$v^2 - u^2 = 2aS, \text{ we get}$$

$$S = 7.11 \times 10^{-2} \text{ m}$$

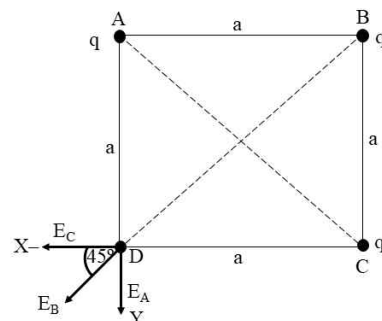
3. (b)

Here, $AB = BC = CD = AD = a$

$$\therefore BD = \sqrt{a^2 + a^2} = \sqrt{2}a$$

$$\text{Now, } E_A = E_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2}$$

$$\text{and } E_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(\sqrt{2}a)^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{2a^2}$$



Since E_A and E_C are equal, their resultant will be equally inclined to them i.e it will along BD .
If is resultant of E_A and E_C them

$$E' = \sqrt{E_A^2 + E_c^2} = \sqrt{E_A^2 + E_A^2} = \sqrt{2} E_A$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\sqrt{2}q}{a^2}$$

Hence, the resultant of electric fields due to the three charges,

$$E = E + E_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sqrt{2}q}{a^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{2a^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{2a^2} (\sqrt{2} + 1)$$

4. (b)

$$q = \epsilon_0 \Phi$$

$$= 8.854 \times 10^{-12} \times 10^3 = 8.854 \times 10^{-9} \text{ C}$$

5. (b)

$$\text{Flue } \Phi = \vec{E} \cdot \vec{A}$$

$$= (5 \times 10^3 \hat{i}) (10 \times 10 \times 10^{-4}) \hat{i}$$

$$= 50 \text{ Nm}^2 \text{ C}^{-1}$$

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CHEMISTRY

CBSE 2021-22 CRASH COURSE

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(SUBJECTIVE QUESTIONS)

(Q. 1 One Marks)

1.

- (i) “Crystalline solids are anisotropic in nature”. What does this statement mean.
- (ii) Classify the following into different categories of crystalline solids :-
Urea , ammonia, tin, graphite, silicon, potassium sulphate, water, argon

(Q. 2 Two Marks)

2. Explain the following terms with suitable examples :

- (i) Schottky defect
- (ii) Frenkel defect.

(Q. 3 Three Marks)

3.

- (i) What do you understand by colligative properties ?
- (ii) State Raoult's law for a binary solution containing volatile components.

(Q. 4 Four Marks)

4. Explain the following with suitable examples:

- (i) Ferrimagnetism
- (ii) Paramagnetism
- (iii) Ferromagnetism
- (iv) F-centre.

(Q. 5 Four Marks)

5.

- (i) Define the following terms :
- Isotonic solutions and hypertonic solutions
 - Van't Hoff factor
 - Plasmolysis
- (ii) What is meant by 'reverse osmosis'?



Answer & Solutions

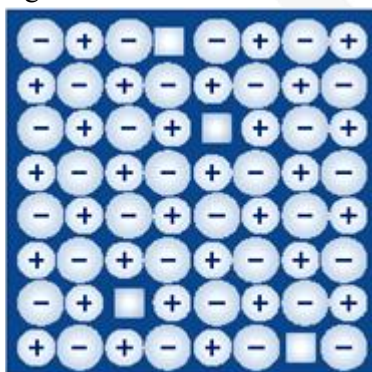
- 1.
- (i) Anisotropic means that properties of crystalline solids like refractive index thermal conductivity electrical conductivity etc. when calculated from different directions comes out to be different. All Crystalline solids are not Anisotropic. Some Crystalline solids are Anisotropic because despite showing periodicity they are not exactly the same in all directions. It all depends on the symmetry of the unit cell of the crystal. They have their atoms arranged in different manner and in three different plane (X,Y,Z).

(ii)

Potassium sulphate	Ionic solid
Benzene	Molecular solid (non-polar)
Urea	Polar molecular solid
Ammonia	Polar molecular solid
Water	Hydrogen bonded molecular solid
Zinc sulphide	Ionic solid
Graphite	Covalent or network solid
Rubidium	Metallic solid
Argon	Non-polar molecular solid
Silicon carbide	Covalent or network solid

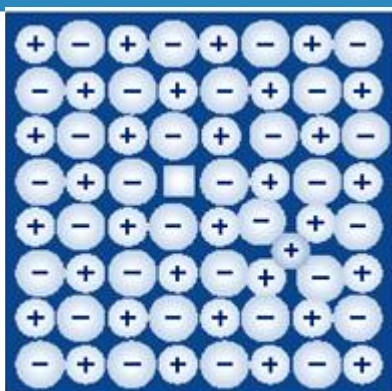
2.

- (i) It is basically a vacancy defect in ionic solids. In order to maintain electrical neutrality, the number of missing cations and anions are equal. Like simple vacancy defect, Schottky defect also decreases the density of the substance. Number of such defects in ionic solids is quite significant.



Schottky defects

- (ii) This defect is shown by ionic solids. In this defect the smaller ions are dislocated from its normal site to an interstitial site. It generates a vacancy defect at its original site and an interstitial defect at its new location. Frenkel defect is also called dislocation defect. In this type of defect, density remains same. Frenkel defect is shown by ionic substance in which there is a large difference in the size of ions, for example, ZnS, AgCl, AgBr and AgI shows this effect due to small size of Zn^{2+} and Ag^+ ions.

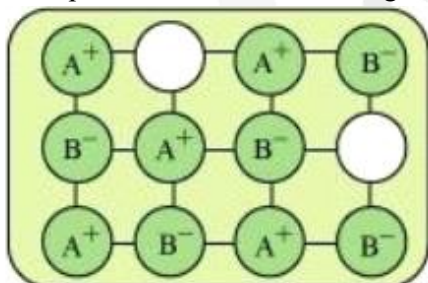


Frenkel defects

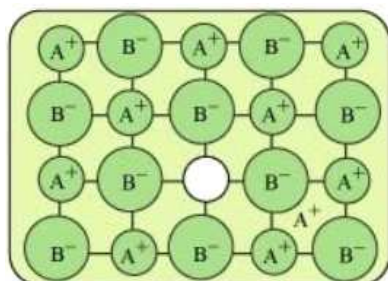
- 3.
- that depend on the ratio of the number of solute particles to the number of solvent molecules in a solution, and not on the nature of the chemical species present. ... The word colligative is derived from the Latin colligatus meaning bound together
 - containing a non-volatile solute is directly related to the mole fraction of solvent (i.e. volatile) in the solution. Relative going-down of vapour pressure is equal to mole fraction of non volatile and non-electrolytic solute.

4.

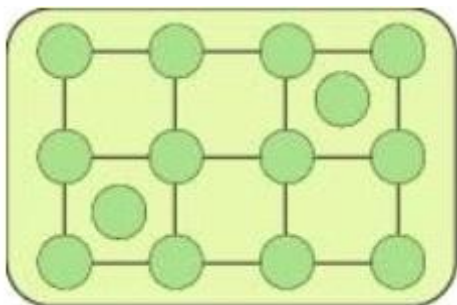
- Schottky defect is basically a vacancy defect shown by ionic solids. In this defect, an equal number of cations and anions are missing to maintain electrical neutrality. It decreases the density of a substance. Significant number of Schottky defects is present in ionic solids. For example, in NaCl, there are approximately 10^6 Schottky pairs per cm^3 at room temperature. Ionic substances containing similar-sized cations and anions show this type of defect. For example: NaCl, KCl, CsCl, AgBr, etc.



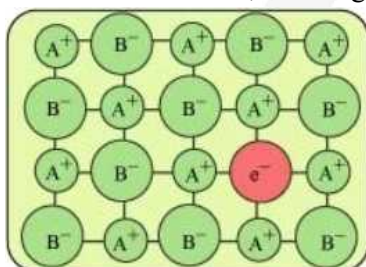
- Ionic solids containing large differences in the sizes of ions show this type of defect. When the smaller ion (usually cation) is dislocated from its normal site to an interstitial site, Frenkel defect is created. It creates a vacancy defect as well as an interstitial defect. Frenkel defect is also known as dislocation defect. Ionic solids such as AgCl, AgBr, AgI and ZnS show this type of defect.



- (iii) Interstitial defect is shown by non-ionic solids. This type of defect is created when some constituent particles (atoms or molecules) occupy an interstitial site of the crystal. The density of a substance increases because of this defect.



- (iv) When the anionic sites of a crystal are occupied by unpaired electrons, the ionic sites are called F-centres. These unpaired electrons impart colour to the crystals. For example, when crystals of NaCl are heated in an atmosphere of sodium vapour, the sodium atoms are deposited on the surface of the crystal. The Cl ions diffuse from the crystal to its surface and combine with Na atoms, forming NaCl. During this process, the Na atoms on the surface of the crystal lose electrons. These released electrons diffuse into the crystal and occupy the vacant anionic sites, creating F-centres.







5.

(i)

- (a) Mole fraction of a component is the ratio of number of moles of the component to the total number of moles of all the components.
- (b) Van't Hoff factor is the ratio of normal molar mass to the abnormal molar mass. Van't Hoff factor is the ratio of observed value of colligative property to calculated value of colligative property assuming no association or dissociation.

(OBJECTIVE QUESTIONS)

1. Which of the following arrangements shows schematic alignment of magnetic moments of antiferromagnetic substances ?

- (a) 
- (b) 
- (c) 
- (d) 

2. Which of the following point defects are shown by AgBr(s) crystals ?

- (A) Schottky defect (B) Frenkel defect
(C) Metal excess defect
(D) Metal deficiency defect

- (a) (A) and (B) (c) (A) and (C)
(b) (C) and (D) (d) (B) and (D)

3. To get an *n*-type semiconductor from silicon, it should be doped with a substance having following valance electrons.

- (a) 2 (c) 3
(b) 1 (d) 5

4. A compound formed by elements X and Y crystallizes in a cubic structure in which atoms X are at the corners of the cube and atoms Y are at the face centres. The formula of the compound is :

- (a) X_3Y (c) XY_2
(b) XY (d) XY_3

5. AB crystallizes in body centred cubic lattice with edge length 'a' equal to 387 pm. The distance between two oppositely charged ions in the lattice is :

- (a) 300 pm (c) 250 pm
(b) 355 pm (d) 200 pm

Answer & Solutions

Q1. (d)

The substances which are expected to possess paramagnetism or ferromagnetism on the basis of magnetic moments of domains but actually they possess zero net magnetic moment are called anti-ferromagnetic substances.

They possess equal no. of domains in opposite directions.

Q2. (a)

Schottky defect is shown by the ionic compounds having comparative size of cation & Anion. Frenkel defect occurs when the cationic size is smaller.

Despite of both the opposite facts, the radius ratio of AgBr being intermediate, makes it a compound showing both defects.

Majorly AgBr shows Frenkel defect.

Q3. (d)

Si has 4 valence electrons when it is doped with a compound having 5 valence electrons, it forms 4 covalent bonds with the doped atom, and the 5th electron is delocalised, increasing the conductivity of silicon and making it negatively charged. This is how n-type semiconductor is formed.

Q4. (d)

corners \rightarrow 8

$$\therefore \frac{1}{8} \times 8 = 1 \text{ atoms (X)}$$

face centres = 6

$$\frac{1}{2} \times 6 = 3 \text{ atoms (Y)}$$

XY_3 is the formula

Q5. (b)

In BCC structure, the cations occupy the corners, whereas the anions occupy the centre of the cube.

\therefore the diagonal of cube $\sqrt{3} \times a$

$$\text{Distance between 2 oppositely charged ions} = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 387 = 335.15 \text{ pm}$$

MATHEMATICS

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(SUBJECTIVE QUESTIONS)

1. Show that the relation R in the set \mathbf{R} of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.
2. Show that the relation R in the set $A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1 in each case.

3. Let $f : \mathbf{N} \rightarrow \mathbf{N}$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

for all $n \in \mathbf{N}$.

State whether the function f is bijective. Justify your answer

4. Consider $f : \mathbf{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{(\sqrt{y+6}) - 1}{3} \right)$.

5. Determine which of the following binary operations on the set \mathbf{R} are associative and which are commutative.

(i) $a * b = 1 \quad \forall a, b \in \mathbf{R}$

(ii) $a * b = \frac{(a+b)}{2} \quad \forall a, b \in \mathbf{R}$



Answer & Solutions

1.

$$R = \{(a, b) : a \leq b^2\}$$

It can be observed that

$$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R, \text{ since } \frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4},$$

$\therefore R$ is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2

$\therefore (4, 1) \notin R$

$\therefore R$ is not symmetric.

Now,

$$(3, 2), (2, 1.5) \in R$$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

2.

$$R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$$

For any element $a \in A$, we have $(a, a) \in R$ as $|a - a| = 0$ is a multiple of 4.

$\therefore R$ is reflexive.

Now, let $(a, b) \in R \Rightarrow |a - b|$ is a multiple of 4.

$\Rightarrow |-(a - b)| = \Rightarrow |b - a|$ is a multiple of 4.

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Now, let $(a, b), (b, c) \in R$.

$\Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

$\Rightarrow (a - b)$ is a multiple of 4 and $(b - c)$ is a multiple of 4.

$\Rightarrow (a - c) = (a - b) + (b - c)$ is a multiple of 4.

$\Rightarrow |a - c|$ is a multiple of 4.

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is $\{1, 5, 9\}$ since

$|1 - 1| = 0$ is a multiple of 4,

$|5 - 1| = 4$ is a multiple of 4, and

$|9 - 1| = 8$ is a multiple of 4.

3.

$$\text{Given } f : \mathbb{N} \rightarrow \mathbb{N} \text{ defined such that } f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

Let $x, y \in \mathbb{N}$ and let them be odd then

$$f(x) = f(y) \Rightarrow \frac{x+1}{2} = \frac{y+1}{2} \Rightarrow x = y$$

If $x, y \in \mathbb{N}$ are both even then also

$$f(x) = f(y) \Rightarrow \frac{x}{2} = \frac{y}{2} \Rightarrow x = y$$

If $x, y \in \mathbb{N}$ are such that x is even and y is odd then

$$f(x) = \frac{x+1}{2} \text{ and } f(y) = \frac{y}{2}$$

Thus, $x \neq y$ for $f(x) = f(y)$

Let $x = 6$ and $y = 5$

$$\text{We get } f(6) = \frac{6}{2} = 3, \quad f(5) = \frac{5+1}{2} = 3$$

$$\therefore f(x) = f(y) \text{ but } x \neq y$$

So, $f(x)$ is not one-one.

Hence, $f(x)$ is not bijective.

4.

Given $f : \mathbb{R} \rightarrow [-5, \infty]$, given by

$$f(x) = 9x^2 + 6x - 5$$

(i) Let $f(x_1) = f(x_2)$

$$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$\Rightarrow 9(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0$$

$$\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$$

$$\Rightarrow x_1 - x_2 = 0 \text{ or } [9(x_1 + x_2) + 6] = 0 \quad \dots(i)$$

$$\Rightarrow x_1 = x_2 \text{ or } 9(x_1 + x_2) = -6 \text{ i.e., } (x_1 + x_2)$$

$$= -\frac{6}{9} \text{ which is not possible}$$

$$\therefore x_1 = x_2$$

$$\text{So, we can say, } f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$\therefore f$ is one-one.

(ii) Let $y \in [-5, \infty]$

So that $y = f(x)$ for some $x \in \mathbb{R}_+$

$$\Rightarrow 9x^2 + 6x - 5 = y$$

$$\Rightarrow 9x^2 + 6x - 5 - y = 0$$

$$\Rightarrow 9x^2 + 6x - (5 + y) = 0 \quad \Rightarrow x = \frac{-6 \pm \sqrt{36 + 4(9)(5 + y)}}{2 \times 9}$$

$$\Rightarrow x = \frac{-6 \pm 6\sqrt{1+5+y}}{18} = \frac{-1 \pm \sqrt{y+6}}{3}$$

$$\Rightarrow x = \frac{-1 + \sqrt{y+6}}{3}, \frac{-1 - \sqrt{y+6}}{3}$$

$$\text{here } x = \frac{-1 + \sqrt{y+6}}{3} \in R_+$$

f is onto

since function is one-one and onto, so it is invertible.

$$f^{-1}(y) = \frac{-1 + \sqrt{y+6}}{3} \quad \text{i.e., } f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$$

5.

(i) Check commutative

*is commutative if

$$a*b = b*a$$

$$\begin{array}{l|l} a*b & b*a \\ = 1 & = 1 \end{array}$$

since

$$a*b = b*a \forall a, b \in R$$

*is commutative

Check associative

*is associative if

$$(a*b)*c = a*(b*c)$$

$$\begin{array}{l|l} (a*b)*c & (a*b)*c \\ = 1*c & = a*1 \\ = 1 & = 1 \end{array}$$

Since $(a*b)*c = a*(b*c) \forall a, b, c \in R$

* is an associative binary operation

(ii) Check commutative

*is commutative if

$$a*b = b*a$$

$$\begin{array}{l|l} a*b & b*a \\ = \frac{a+b}{2} & = \frac{b+a}{2} \\ & = \frac{a+b}{2} \end{array}$$

Since

$$a*b = b*a \forall a, b \in R$$

*is commutative

Check associate

*is associative if

$$(a*b)*c = a*(b*c)$$

$(a*b)*c$ $= \left(\frac{a+b}{2}\right)*c$ $= \frac{\frac{a+b}{2} + c}{2}$ $= \frac{a+b+2c}{2}$ $= \frac{a+b+2c}{4}$	$a*(b*c)$ $= a*\left(\frac{b+c}{2}\right)$ $= a + \frac{b+c}{2}$ $= \frac{2a+b+c}{2}$ $= \frac{2a+b+c}{4}$
--	--

Since $(a*b)*c \neq a*(b*c) \forall a, b, c \in \mathbb{R}$

* is not an associative binary operation



(OBJECTIVE QUESTIONS)

- Which of the following functions are equal
 - $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1}x)$
 - $\frac{x^2-4}{x-2}$, $x+2$
 - $\frac{x^2}{x}$, x
 - $A=\{1,2\}$, $B=\{3,6\}$
 $f:A\rightarrow B$ given by $f(x) = x^2+2$ and
 $g:A\rightarrow B$ given by $g(x) = 3x$
- Let $f : \left[\frac{1}{2}, \infty\right) \rightarrow \left[\frac{3}{4}, \infty\right)$, where $f(x) = x^2 - x + 1$ is
 - one-one onto
 - many one-into
 - many one-onto
 - one-one into
- IF $A = \{1,2,3\}$
 $B = \{4,5,6,7\}$ and
 $f = \{(1,4) (2,5) (3,6)\}$ is a function from A to B then f is
 - one-one
 - onto
 - many one
 - both (a) and (b)
- The range of the function $f(x) = \frac{|x-2|}{x-2}$, $x \neq 2$ is
 - $\{1,0,-1\}$
 - $\{1\}$
 - $\{1,-1\}$
 - None of these
- $f:\mathbb{R}\rightarrow\mathbb{R}$ and $g:\mathbb{R}\rightarrow\mathbb{R}$ are given by $f(x) = |x|$ and $g(x) = |5x-2|$, then $f\circ g$ is
 - $|5x-2|$
 - $5x-2$
 - $2-5x$
 - None of these

Answer & Solutions

1. (d)

(a) $\sin^{-1}(\sin x) \neq \sin(\sin^{-1}x)$

 \therefore Those functions are equal

whose range & domain are equal

But $\sin^{-1}(\sin x) = x \Rightarrow x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

But $\sin(\sin^{-1}x) = x \Rightarrow x \in [-1, 1]$

(b) $\frac{(x^2-4)}{(x-2)} = x+2, x \neq 2$

Domain of $\frac{x^2-4}{x-2} \Rightarrow \mathbb{R} - 2$

& Domain of $x+2$ is \mathbb{R}

Hence they are not equal

(c) same explanation as (b)

(d) $A = \{1, 2\}, B = \{3, 6\}$

$f(1) = 3$

$f(2) = 6$

$g(x) = 3x$

$g(1) = 3$

$g(2) = 6$

Since range & domain in both functions is equal.

Hence functions are equal function

2. (a)

$f(x) = x^2 - x + 1$

$f: \left(\frac{1}{2}, \infty\right) \rightarrow \left(\frac{3}{4}, \infty\right)$

For one-one

$f(x_1) = f(x_2)$

$x_1^2 - x_1 + 1 = x_2^2 - x_2 + 1$

$x_1^2 - x_2^2 - x_1 + x_2 = 0$

$(x_1 - x_2)(x_1 + x_2) - 1(x_1 - x_2) = 0$

$(x_1 - x_2)(x_1 + x_2 - 1) = 0$

either $x_1 = x_2$ or $x_1 + x_2 = 1$

But $x_1 + x_2 = 1$

only when $x_1 = x_2$

& for no other value

$\therefore x_1 = x_2$

Hence one-one

onto

$$\begin{aligned} f(x) &= x^2 - x + 1 \\ &= x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 \\ &= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \end{aligned}$$

[Using completing the square method]

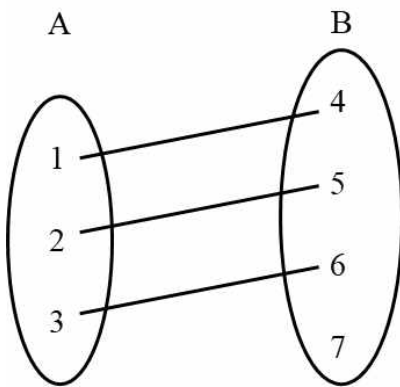
$$\text{For } x \geq \frac{1}{2}$$

$$y \geq \frac{3}{4}$$

Hence range = Codomain

Hence function is onto

3. (a)

Clearly f is one but not onto

4. (c)

$$f(x) = \frac{|x-2|}{x-2}, x \neq 2 \text{ is}$$

$$f(x) = \begin{cases} 1 & x-2 > 0 \\ -1 & x-2 < 0 \end{cases}$$

 \therefore Range is $\{1, -1\}$

5. (a)

$$f(x) = |x|$$

$$g(x) = |5x - 2|$$

$$f(g(x)) = ||5x - 2||$$

$$= |5x - 2|$$