CRASH COURSE

JEE MAIN 2021-22

PHYSICS

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SET-1

1. Force F is given in terms of time t and distance x by

$$F = A \sin Ct + B \cos Dx$$

Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by

(a) $[MLT^{-2}, M^0L^0T^{-1}]$
(b) $[MLT^{-2}, M^0L^{-1}T^0]$
(c) $[M^0L^0T^0, M^0LT^{-1}]$
(d) $[M^0LT^{-1}, M^0L^0T^0]$

2. What are the dimensions of electrical resistance?
(a) $ML^2T^{-2}I^2$
(b) $[ML^2T^{-3}I^{-2}]$
(c) $[ML^2T^{-3}I^2]$
(d) $[ML^2T^{-2}I^{-2}]$

3. From some instruments, current measured is i = 10.0 Amp, potential difference measured is V = 100.0 V, length of wire is 31.4 cm, and diameter of wire is 2.00 mm (all in correct significant figures) will be (Use π = 3.14)
   (a) 1.00×10^{-4} Ω-m
   (b) 1.0×10^{-4} Ω-m
   (c) 1×10^{-4} Ω-m
   (d) 1.000×10^{-4} Ω-m

4. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is
   (a) (0.34 ± 0.02) cm
   (b) (0.17 ± 0.02) cm
   (c) (0.17 ± 0.01) cm
   (d) (0.34 ± 0.01) cm

5. 1 cm on the main scale of a vernier callipers is divided into 10 equal. If 10 divisions of vernier coincide with B small divisions of main scale, then the least count of the vernier calliper is
   (a) 0.01 cm
   (b) 0.02 cm
   (c) 0.05 cm
   (d) 0.005 cm

6. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction heat is produced. The expression for the rate of production of heat when the sphere has acquired the terminal velocity is

   (a) $\left[\dfrac{8\pi g}{27\eta}(\rho-\sigma)^2\right]r^5$
   (b) $\left[\dfrac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right]r^5$
   (c) $\left[\dfrac{8\pi g^2}{27\eta}(\rho-\sigma)\right]r^5$
   (d) $\left[\dfrac{8\pi g^2}{27\eta^2}(\rho-\sigma)\right]r^5$

SET-2

1. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then, x_1 ; x_2 ; x_3 is

   (a) 1 : 2 : 4
   (b) 1 : 2 : 5
   (c) 1 : 3 : 5
   (d) 1 : 3 : 9

2. A ball is dropped from the top of a building. They ball takes 0.5 s to fall past the 3 m length of a window some distance from the top of the building. If the speed of the ball at the top and at the bottom of the window are v_T and v_B respectively, then
   (a) $v_T + v_B = 12\ \text{ms}^{-1}$
   (b) $v_T - v_B = 4.9\ \text{ms}^{-1}$
   (c) $v_B + v_T = 1\ \text{ms}^{-1}$
   (d) $\frac{v_B}{v_T} = 2$

3. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in the last two seconds is
   (a) $\frac{2v(n-1)}{n}$
   (b) $\frac{v(n-1)}{a}$
   (c) $\frac{v(n+1)}{n}$
   (d) $\frac{2v(2n+1)}{a}$

4. A person walks up a stationary escalator in time t_1. If he remains stationary on the escalator, then it can take him up in time t_2. How much time would it take him to walk up the moving escalator?
   (a) $\frac{t_1+t_2}{2}$
   (b) $\sqrt{t_1 t_2}$
   (c) $\frac{t_1 t_2}{t_1+t_2}$
   (d) t_1+t_2

Answer & Solutions

SET-1

1. (c)

$$\frac{A}{B} = M^0L^0T^0 \qquad \frac{C}{D} = \frac{X}{t} = LT^{-1}$$

2. (b)

3. (a)

$$s = \frac{\pi D^2}{4L}.\frac{V}{I}$$

$$\Rightarrow \frac{3.14 \times (2 \times 10^{-3})^2}{4(.314)}\left(\frac{100.0}{10.0}\right)$$

$s = 1.00 \times 10^{-4}$ Ω-m

4. (c)

$(R_1 \pm \Delta R_1) = (2.11 \pm 0.005)$ cm

$(R_2 \pm \Delta R_2) = (1.945 \pm 0.005)$ cm

Thickness $t = R_1 - R_2 = 0.17$ cm

$\Delta t = \Delta R_1 + \Delta R_2 = 0.01$ cm

$(t \pm \Delta t) = (0.17 \pm 0.01)$ cm

5. (b)

10 VSD = 8 MSD

1 VSD = 0.8 MSD

LC = 1 MSD – 1 VSD

= 1 MSD – 0.8 MSD

$= 0.2 \text{ MSD} = 0.2 \times \frac{1}{10}$ cm

= 0.02 cm

6. (b)

Rate of heat production = – (Power of viscous force)

$\frac{dH}{dt}$ = (viscous force) (terminal velocity)

$= (6\pi\eta rv)(v)$ ...(i)

$$v = \frac{2}{9}\frac{(s-\sigma)r^2g}{\eta}$$

$$\frac{dx}{dt} = \left[\frac{8\pi g^2}{2\eta}(s-\sigma)\right] r^5$$

SET-2

1. (c)

$$x_1 = \frac{1}{2}a(10)^2 = 50a$$

$$x_2 = \frac{1}{2}a(20)^2 - \frac{1}{2}(a)(10)^2 = 150a$$

$$x_3 = \frac{1}{2}a(30)^2 - \frac{1}{2}a(20)^2 = 250a$$

$x_1 : x_2 : x_3 = 1 : 3 : 5$

2. (a)

$v_B = v_T + gt$

$= v_T + 9.8 \times 0.5$

$= v_B - v_T = 4.9$

$v_B^2 - v_T^2 = 58.8$

Solving $v_T + v_B = 12\ ms^{-1}$

3. (a)

$v = an \quad \Rightarrow a = v/n$

$$s = \frac{1}{2}a(n)2 - \frac{1}{2}a(n-2)^2$$

$= v/2n\ [4n - 4]$

$$= \frac{2v(n-1)}{n}$$

4. (c)

Solution : Speed of man w.r.t. escalator $v_{mc} = L/t_1$

speed of escalator $v_c = L/t_2$

speed of man w.r.t. grand

$$v_m = v_{mc} + v_c = L\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

$$\text{time } t = \frac{L}{v_m} = \frac{t_1 t_2}{t_1 + t_2}$$

CHEMISTRY

JEE Main 2021-22 CRASH COURSE

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1. 5.85g NaCl is dissolved in 1 L water. The number of ions of Na^+ and Cl^- is 1mL of this solution will be :
   (a) 6.02×10^{19}
   (b) 1.2×10^{22}
   (c) 1.2×10^{20}
   (d) 6.02×10^{20}

2. If mass of neutron is assumed to half of its original value, whereas that of proton is assumed to be twice of its original value then the atomic mass of ${}^{14}_{6}C$ will be:
   (a) same
   (b) 14.28% more
   (c) 14.28% less
   (d) 28.56% less

3. The density of a liquid is 1.2 g/mL. There are 35 drops in 2mL. The number of molecules in 1 drop is (molecular weight of liquid = 70).
   (a) $\frac{1.2}{35}N_A$
   (b) $\left(\frac{1}{35}\right)^2 N_A$
   (c) $\frac{1.2}{(35)^2}N_A$
   (d) 1.2 N_A

4. 26.8 gm of $Na_2SO_4.nH_2O$ contains 12.6 gm of water. The value of '*n*' is :
   (a) 1
   (b) 10
   (c) 6
   (d) 7

5. How many moles of Na^+ ions are in 20mL of 0.4M Na_3PO_4 ?
   (a) 0.008
   (b) 0.024
   (c) 0.05
   (d) 0.20

6. In the reaction ;

$$CO + \frac{1}{2} O_2 \rightarrow CO_2; \quad N_2 + O_2 \rightarrow 2NO$$

   10mL of mixture containing carbon monoxide and nitrogen required 7mL oxygen to form CO_2 and NO, on combustion. The volume of N_2 in the mixture will be :
   (a) 7/2mL
   (b) 17/2mL
   (c) 4mL
   (d) 7mL

7. A mixture of ethane and ethene occupies 40 litre at 1.00 atm and at 400K. The mixture reacts completely with 130g of O_2 to produce CO_2 and H_2O. Assuming ideal gas behaviour, calculate the mole fractions of C_2H_6 and C_2H_4 in the mixture.
   (a) 11.12, 12.13
   (b) 18.34, 34.36
   (c) 66.66, 33.34
   (d) 35.34, 31.34

8. A mixture of HCOOH and $H_2C_2O_4$ is heated with concentrated H_2SO_4. The gas produced is collected and on treating with KOH solution, the volume of the gas decreases by 1/6th. Calculate the molar ratio of the two acids in the original mixture.
   (a) 4:1
   (b) 3:4
   (c) 4:4
   (d) 2:1

9. 3.6g mixture of sodium chloride and potassium chloride is dissolved in water. The solution is treated with excess of silver nitrate solution. 7.74g of silver chloride is obtained. Find the percentage of sodium chloride and potassium chloride in the mixture.
   (a) 52.2, 48.2
   (b) 42.7, 57.3
   (c) 58.2, 65.4
   (d) 38.5, 55.1

10. A mixture in which the mole ration of H_2 and O_2 is 2:1, is used to prepare water by the reaction :

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$

The total pressure in the container is 0.8 atm at 20°C before the reaction. Determine the final pressure at 120°C after the reaction, assuming 80% yield of water.

(a) 0.742
(b) 2.423
(c) 1.256
(d) 0.787

Answer & Solutions

1. (c)

No. of moles of NaCl $= \frac{5.85}{58.5} = 0.1$

$1 \text{ NaCl} \rightarrow (Na^+ + Cl^-) = 2$ ion

$\therefore$ $1 \text{ mole NaCl} \rightarrow 2 \times 6.022 \times 10^{23}$ ions

$= \frac{1.2 \times 10^{23}}{1000}$ ion

$= 1.2 \times 10^{20}$ ions

2. (b)

${}^{14}_{6}C \rightarrow$ no of neutrons = 8

no of Protons = 6

new atomic mass $= \frac{1}{2} \times 8 + 2 \times 6$

$= 4 + 12 = 16$

original atomic mass = 14

% increase $= \frac{16-14}{14} \times 100 = \frac{2}{14} \times 100$

$= \frac{100}{7} = 14.28\%$

3. (c)

$\delta = 1.2$ g/ml

volume of one drop $= \left(\frac{2}{35}\right)$

density $= \frac{\text{mass}}{\text{volume}}$

$1.2 = \frac{W}{\left(\frac{2}{35}\right)}$

$1.2 \times \left(\frac{2}{35}\right) = W$

$\Rightarrow$ $1.2 \times \left(\frac{2}{35}\right) = nM \rightarrow 70$

$n = \frac{1.2}{(35)^2}$

no of molecules $= \frac{1.2}{(35)^2} \times 6.022 \times 10^{23}$

no of molecules $= \frac{1.2}{(35)^2} \times NA$

4. (d)

molecular mass of $Na_2So_4.nH_2O$ = (142 + 18n)

(142 + 18n) $Na_2So_4.nH_2O \rightarrow 18n\ H_2O$

26.8 gm $Na_2So_4.nH_2O \rightarrow \frac{18n}{(142+18n)} \times 26.8$

Now, $\frac{18n}{(142+18n)} \times 26.8 = 12.6$

$\Rightarrow \frac{12.6}{26.8} = \frac{18n}{(142+18n)}$

$\Rightarrow n = \frac{142}{18} \approx 7.88 \approx 7$

5. (b)

1 mole $Na_3PO_4 \rightarrow$ 3 mole Na^+

$\left(\frac{1}{125}\right)$ mole $Na_3PO_4 \rightarrow \left(\frac{3}{125}\right)$ mole of Na^+

= 0.024

6. (c)

$CO + \frac{1}{2}O_2 \rightarrow CO$

$x \qquad (x/2)$

$N_2 + O_2 \rightarrow 2NO$

$y \qquad y$

$x + y \quad ; \qquad \frac{x}{2} + y = 7$

$x + 2y = 14$

$x + y = 10$

$x + 2y = 14$

$y = 4 \quad \Rightarrow \quad y = 4$

Volume of N_2 in the mixture = 4

7. (c)

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{1 \times V_1}{2+3} = \frac{1 \times 40}{400}$

$V_1 = \frac{2+3}{10} = 27.3$ litre

Mixture $\rightarrow C_2H_6$, C_2H_4

Let

the volume of C_2H_6 = x litre

the volume of C2H4 = (27.3 – x) litre

$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$ + Energy

$$1 \text{ vol} \quad \frac{7}{2}\text{vol}$$

$$x \text{ vol} \quad \frac{7}{2}x \text{ vol}$$

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O + \text{Energy}$$

$$1 \text{ vol} \quad 3 \text{ vol}$$

$$(27.3 - x) \quad 3(27.3 - x)$$

Total volume of oxygen required

$$\frac{7}{2}x + 3(27.3 - x)$$

Now,

$$\text{mass of oxygen} = 130$$

$$\therefore \quad \left\{\frac{7}{2}x + 3(27.3 - x)\right\} \times \frac{32}{22.4} = 130$$

$$x = 18.2$$

$$\text{mole fraction } (C_2H_6) = \frac{18.2}{27.3} \times 100$$

$$\approx 66.66\%$$

$$\text{mole fraction } (C_2H_4) = 100 - 66.66\%$$

$$= 33.34\%$$

8. (a)

$$\underset{(a)}{HCOOH} \xrightarrow{\text{Conc.}H_2SO_4} \underset{(a)}{CO} + H_2O$$

$$\underset{(b)}{\begin{array}{l} COOH \\ | \\ COOH \end{array}} \xrightarrow{\text{Conc.}H_2SO_4} \underset{(b)}{CO} + \underset{(b)}{Co_2} + H_2O$$

Total no. of m oles of gas = a + b + b

= a + 2b

on treating KOH solution, Co_2 gas is absorbed

$$b = \frac{a + 2b}{6}$$

$$\Rightarrow \quad 6b = a + 2b$$

$$\Rightarrow \quad \frac{a}{b} = \frac{4}{1}$$

9. (b)

let the mass of NaCl = x gm

mass of KCl = $(3.6 - x)$gm

$$\left.\begin{array}{l} NaCl \\ KCl \end{array}\right] AgNo_3 \longrightarrow \underset{7.74 \text{ gm}}{\overset{AgCl}{\downarrow}}$$

$$NaCl + AgNo_3 \longrightarrow AgCl + NaNO_3$$

58.5 gm ↓ … ↓ 143.5 gm

$$58.5 \text{ gm } NaCl \longrightarrow 143.5 \text{ gm } AgCl$$

$$x \text{ gm } NaCl \longrightarrow \frac{143.5}{58.5}\times x \text{ gm } AgCl$$

$$KCl + AgNo_3 \longrightarrow AgCl + KNo_3$$

74.5 gm   143.5 gm

$$74.5 \text{ gm } KCl \longrightarrow 143.5 \text{ gm } AgCl$$

$$(3.6 - x) \text{ gm } KCl \longrightarrow \frac{143.5}{74.5}\times(31.6 - x) \text{ gm } AgCl$$

$$\text{Total AgCl} = \frac{143.5}{58.5}\times x + \frac{143.5}{74.5}(3.6 - x)$$

$$7.74 = \frac{143.5}{58.5}\times x + \frac{143.5}{74.5}(3.6 - x)$$

$x = 1.54$

$$\% \text{ NaCl} = \frac{1.54}{3.6}\times 100 = 42.7\%$$

$$\% \text{ KCl} = 100 - 42.7 = 57.3\%$$

10. (d)

$$2H_2 + O_2 \longrightarrow 2H_2O$$

$2H_2$	O_2	$2H_2O$
2n	n	O
$(2n - 2x)$	$(n - x)$	$2x$

Now, $\frac{2x}{2n}\times 100 = 80$

$$\frac{x}{n} = \frac{80}{100} = 0.8$$

$x = 0.8$ n

moles of H_2 $= 2n - 2x$
$= 2n - 2 \times 0.8$ n
$= 2n - 1.6$ n
$= 0.4$ n

moles of O_2 $= n - x$
$= n - 0.8$ n
$= 0.2$ n

moles of H_2O $= 2\times 0.8$ n
$= 1.6$ n

Total no of moles at final = 0.4n + 0.2n + 1.6n

Total no of moles before the react = 2n + n + 0 = 3n

Now,

$PV = nRT$

$(0.8) \times V = 3n \times R \times 293$ ...(i)

$P \times V = 2.2n \times R \times 393$ ...(ii)

eq (i) divided eq (ii)

$$\frac{0.8 \times V}{P \times V} = \frac{3n \times R \times 293}{2.2n \times R \times 393}$$

$$\Rightarrow \quad \frac{0.8}{P} = \frac{3 \times 293}{2.2 \times 393} = P = \frac{2.2 \times 393 \times 0.8}{3 \times 293}$$

$P = 0.787$

MATHEMATICS

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1. The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ & the plane $x - y + z = 16$ is
(a) 8
(b) $3\sqrt{21}$
(c) 13
(d) $2\sqrt{14}$

2. The distance of the point (1,–5,9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is
(a) $3\sqrt{10}$
(b) $10\sqrt{3}$
(c) $\frac{10}{\sqrt{3}}$
(d) $\frac{20}{3}$

3. If the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane $lx + my - z = 9$ them $l^2 + m^2$ is equal to
(a) 26
(b) 18
(c) 5
(d) 2

4. If the image of the point $(1, -2, 3)$ in the plane $2x + 3y - 4z + 22 = 0$ measured parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q them PQ is equal to

(a) $\sqrt{42}$
(b) $6\sqrt{5}$
(c) $3\sqrt{5}$
(d) $2\sqrt{42}$

5.

(i) The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane $x + y + z = 7$ is

(a) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{3}$

(b) $\frac{2}{3}$ (d) $\frac{\sqrt{2}}{\sqrt{3}}$

(ii) Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(a) $x + 2y - 2z = 0$
(b) $3x + 2y - 2z = 0$
(c) $x - 2y + z = 0$
(d) $5x = 2y - 4z = 0$

6.

(i) Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ & $\vec{a}.\vec{c} = 4$ them $|\vec{c}|^2 = ?$

(ii) Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\,\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\,\hat{k}$ & $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ be three vectors such that the projection of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$ then $|\vec{b}| = ?$

7.

(i) Let A be a point on the line $\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ & B(3, 2, 6) be a point in space then value of μ for which $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is

(ii) Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ & $\vec{b}$ are non – collinear. The value of λ for which $\vec{a}$ & $\vec{b}$ are collinear is

8.

(i) Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ & $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ & $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors then, the non zero vector $\vec{a} \times \vec{c}$ is

(ii) Let $\sqrt{3}\,\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\,\hat{j}$ & $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of points A, B, C with respect to origin O. If the distance of C from the bisector of the acute angle between OA & OB is $\frac{3}{\sqrt{2}}$ then sum of all possible values of β is

(iii) Let $\vec{a}, \vec{b}, \vec{c}$ be 3 unit vectors out of which vectors $\vec{b}$ & $\vec{c}$ are non parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $|\alpha - \beta|$ is equal to :

9. Let $[x]$ denote the greatest integer less than or equal to x. Then,
$$\lim_{x\to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2} :$$
(a) Does not exist
(b) equals π
(c) equals 1 + π
(d) equals 0

10.

(i) $$\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4} \text{ is :}$$

(a) exists and equals $\frac{1}{4\sqrt{2}}$

(b) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) Does not exist

Answer & Solutions

1. (c)

Eqn. of line : $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12} = \lambda$

$\therefore$ Any point on this line is

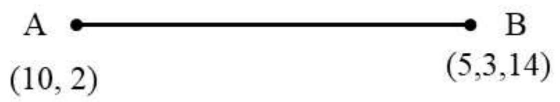
$(x,y,z) = (3\lambda + 2,\ 4\lambda - 1,\ 12\lambda + 2)$

Let (x,y,z) lies on the plane x – y + z = 16

$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 16$

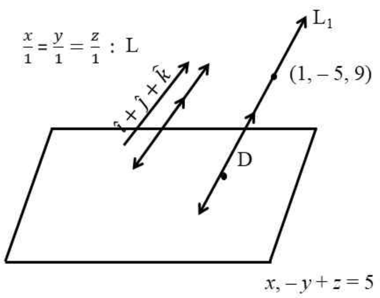
$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection of line & plane is (5, 3, 14)



$AB = \sqrt{16+9+144} = 13$ units

2. (b)



Eq^n. of L_1 : $\dfrac{x-1}{1} = \dfrac{y+5}{1} = \dfrac{z-9}{1} = \lambda$

$\therefore (x,y,z) \equiv (\lambda + 1,\ \lambda - 5,\ \lambda + 9)$.

Let $(\lambda + 1,\ \lambda - 5,\ \lambda + 9)$ lie on the plane

$\therefore (\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$

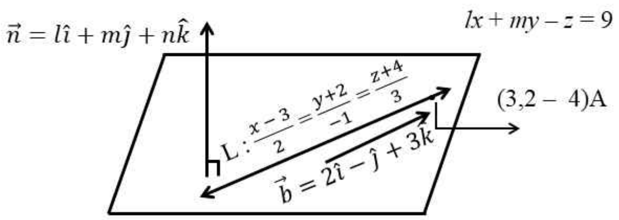
$\lambda = -10$

$\therefore D \equiv (-9, -15, -1)$

And, $AD = \sqrt{100+100+100}$

$= 10\sqrt{3}$ unit

3. (d)



D.R of plane are l, m – 1

Now, A(3, – 2,4) lies on the plane

$\therefore l(3) + m(-2) - (-4) = 9$

$3l - 2cm = 5$ (1)

& $\vec{n} \perp \vec{b} \Rightarrow \vec{n}.\vec{b} = O$

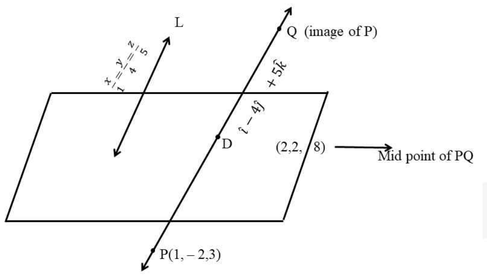
i.e. $2l - m = 3$ (2)

Solving (1) & (2) we get

$l = 1, m = -1$

$\therefore l^2 + m^2 = 2$

4. (d)



Eq^n. of line passing through P harting

D. R 1,4,5 is : $\dfrac{x-1}{1} = \dfrac{y+2}{4} = \dfrac{z-3}{5}$

$\therefore (x,y,z) \equiv (\lambda + 1, 4\lambda - 2, 5\lambda + 3)$ lies on the plane

$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

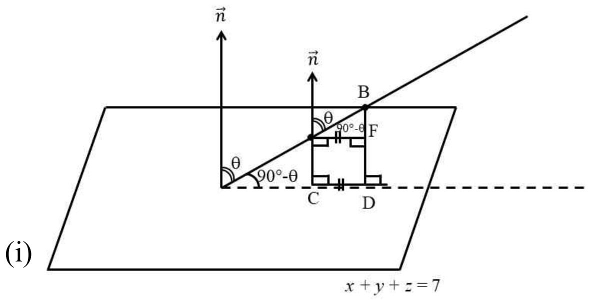
$\Rightarrow \lambda = 1$

$\therefore D \equiv (2,2,8)$

$\therefore PQ = PD + DQ = 2PD$

$2\sqrt{1^2 + 4^2 + 5^2} = 2\sqrt{42}$

5. (i) → (d) (ii) → (c)



A ≡ (5, – 5, 4) B ≡ (4, –1,3)

$\overrightarrow{AB} = (4\hat{i} - \hat{j} + 3\hat{k}) - (5\hat{i} - \hat{j} + 4\hat{k}) = -\hat{i} + 0\hat{j} - \hat{k}.$

$\vec{n}.\overrightarrow{AB} = |\vec{n}||\overrightarrow{AB}|\cos\theta$

$(\hat{i} + \hat{j} + \hat{k}).(-\hat{j} + 0\hat{j} - \hat{k}) = \sqrt{3}\,\sqrt{2}\cos\theta$

$\Rightarrow \cos\theta = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \sin\theta = 1/\sqrt{3}$

Projection = AF

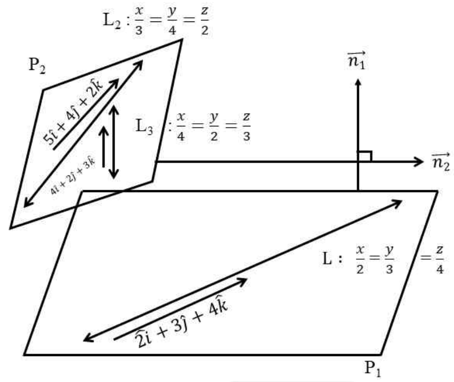
$\cos(90^\circ - \theta) = \dfrac{AF}{AB}$

$\Rightarrow$ AF= AB cos $(90^\circ - \theta)$

$= |\overrightarrow{AB}| \sin\theta$

$= \sqrt{2} \times \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}$

(ii)



Now, $P_1 \perp P_2$

$\Rightarrow \overrightarrow{n_1} \perp \overrightarrow{n_2}$

& $\overrightarrow{n_1} \perp (2\hat{i} + 3\hat{j} + 4\hat{k})$ & $\overrightarrow{n_2} \perp (4\hat{i} + 2\hat{j} + 3\hat{k})$

& $\overrightarrow{n_2} \perp (3\hat{i} + 4\hat{j} + 2\hat{k})$ $\overrightarrow{n_2} \perp (4\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{n_2} = (3\hat{i} + 4\hat{j} + 2\hat{k}) \times (4\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{n_2} = 8\hat{i} - \hat{j} - 10\hat{k}$

& $\overrightarrow{n_1} = \overrightarrow{n_2} = 0 \Rightarrow 8a - b10c = 0$

& $\overrightarrow{n_1} . (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0 \Rightarrow 2a + 3b + 4c = 0$

By (1) & (2),

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{1}$$

$\therefore$ Eq^n of plane is $x - 2y + z = 0$

6. (i) $\rightarrow$ 19/2 (ii) $\rightarrow$ 6

(i) $\dfrac{\vec{a}}{\vec{\ell}} = \dfrac{\hat{i} - \hat{j}}{\hat{i} + \hat{j} + \hat{k}}$

$(\vec{a} \times \vec{c}) + \vec{b} = \vec{O}$

$\Rightarrow \vec{a} \times \vec{c} = \overrightarrow{-b}$

$(\vec{a} \times \vec{c}) \times \vec{a} = (-\vec{b}) \times \vec{a}$

$(\vec{a}.\ \vec{b})\ \vec{c} - (\vec{c}.\ \vec{a})\ \vec{a} = -(\vec{b} \times \vec{a})$

$$|\vec{a}|^2\ \vec{c} - 4\,\vec{a} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$= -\hat{i} - \hat{j} + 2\hat{k}$

$2\vec{c} - 4\vec{a} = (-\hat{i} - \hat{j} + 2\hat{k})$

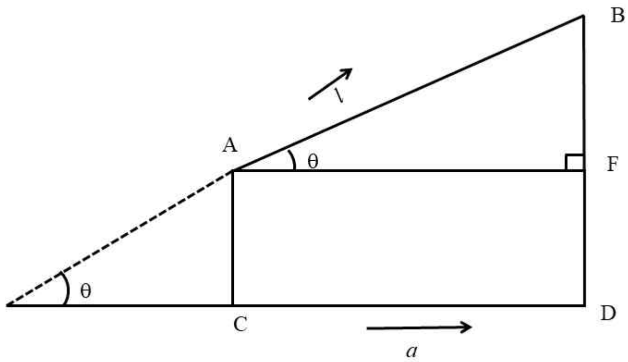
$2\vec{c} = (-\hat{i} - \hat{j} + 2\hat{k}) + (4\hat{i} - 4\hat{j} + 0\hat{k})$

$2\vec{c} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$$\vec{c} = \frac{3}{2}\hat{i} - \frac{5}{2}\hat{j} + \frac{2}{2}\hat{k}$$

$$|\vec{c}|^2 = (3/2)^2 + \left(\frac{-5}{2}\right)^2 + (1)^2$$
$$= 19/2$$

(ii)



$$\cos\theta = \frac{Af}{|\vec{\ell}|} \Rightarrow Af = |\vec{\ell}|\cos\theta = \frac{|\vec{a}|\vec{a}.\vec{a}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$$

$$\therefore \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$$

So, $\vec{a}.\vec{b} = |\vec{a}|^2$

$\therefore$ $b_1 + b_2 + 2 = 4$

$b_1 + b_2 = 2$ (1) $(\because |\vec{a}| = 2)$

$\vec{a} + \vec{b} = (1 + b_1)\hat{i} + (1 + b_2)\hat{j} + (2\sqrt{2})\hat{k}\ (\vec{a} + \vec{b}).\vec{c} = 0$

$\Rightarrow 12 + 4b_1 = 0$

$\Rightarrow b_1 = -3$

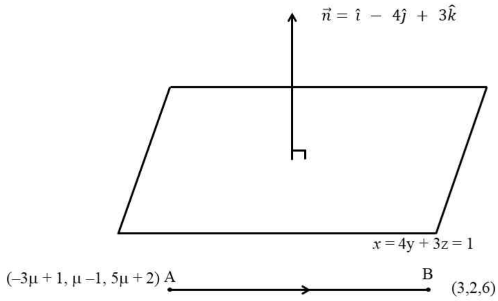
$b_2 = 5$

$\therefore$ $\vec{a} = -3\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$|\vec{a}| = \sqrt{9 + 25 + 2} = \sqrt{36} = 6$

7. (i) $\rightarrow \mu = 1/4$. (ii) $\rightarrow \lambda = -4$.

(i)

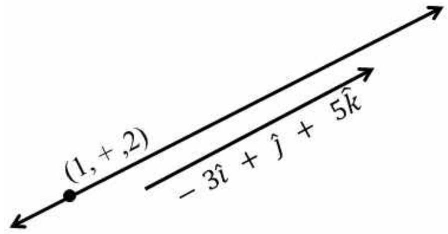


$\vec{n} \perp \overrightarrow{AB}$

$\Rightarrow$ $\vec{n}.\overrightarrow{AB} = O$ (1)

$\Rightarrow$ $(\hat{i} - 4\hat{j} + 3\hat{k}).\overrightarrow{AB} = O$

Let $\vec{r} = (1-4\mu)\hat{i} + (\mu-2)\hat{j} + (2+\mu)\hat{k}$
$\underbrace{(\hat{i}-\hat{j}+2\hat{k})}_{\vec{a}} + \mu\underbrace{(-3\hat{i}+\hat{j}\,5\hat{k})}_{\vec{b}}$



L : $\dfrac{x-1}{-3} = \dfrac{y+1}{1} = \dfrac{z-2}{5} = \mu$

(x,y,z) = (− 3μ + 1, μ − 1, 5μ + 2)

∴ $\overrightarrow{AB}$ = (3μ + 2) $\hat{i}$ + (3 − μ)$\hat{j}$ + (4 − 5μ) $\hat{k}$

By(1)

μ = 1/4.

(ii) ∃ constant x ;

$\alpha = x\beta$

$(\lambda - 2)\vec{a} + \vec{b} = x\,((4\lambda-2)x)\ \vec{a} + 3x\ \vec{b}$

$= ((4\lambda-2)x)\ \vec{a} + 3x\ \vec{b}$

$\Rightarrow \underbrace{((\lambda-2)-(4\lambda-2)x)}_{k_1}\vec{a} + \underbrace{(1-3x)}_{k_2}\vec{b} = 0$

$\Rightarrow 1 - 3x = 0 \Rightarrow 1 = x$ or $x = 1/3$

& $\lambda - 2 - (4\lambda-2)x = 0$

$\Rightarrow \lambda = -4.$

8. (i) → $10\hat{i}+5\hat{j}$ (ii) → 2β = − 2 i.e. β = 1. (iii) → ∴ $|\alpha-\theta| = |\pi|2-\pi|3| = 30^\circ$

(i) $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^{2-}1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^{2-}\ 9\lambda + 18 = 0$ (i)

$\lambda - 2$ is a factor of (i)

∴ $\lambda^3 - 2\lambda^2 - 9\lambda + 18\ (\lambda-2)\ (\lambda^2-9) = 0$

i.e. $(\lambda-2)(\lambda-3)(\lambda+3) = 0$

Values of λ are 2,3, − 3

$\vec{c} = 2\vec{i} + 4\hat{j} + 3\hat{k}$ or

$\vec{c} = 2\vec{i} + 4\hat{j} + 8\hat{k}$

$= 2\vec{a}$

∴ $\vec{a}$ & $\vec{c}$ are colinear vectors

$\Rightarrow \vec{a} \times \vec{c} = \vec{0}$

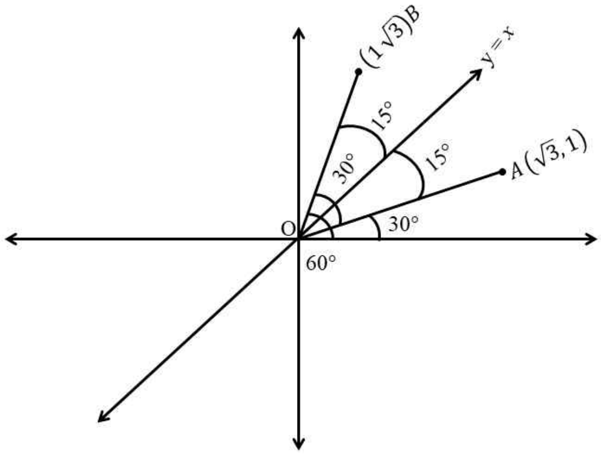
∴ $\lambda \neq 3, -3.$

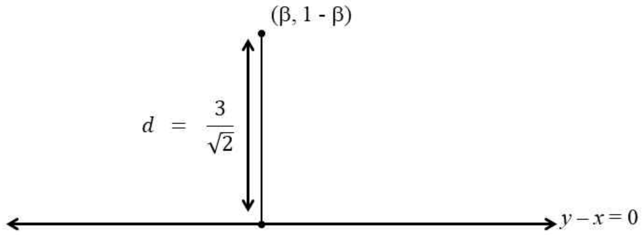
$\lambda = 2$

& $\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

∴ $\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$

(ii)





$$\frac{3}{\sqrt{2}} = \mathrm{d} = \frac{|1-\beta-\beta|}{\sqrt{1^2+1^2}}$$

$\Rightarrow |1-2\beta| = 3$

$\Rightarrow 1 - 2\beta = \pm 3$

$\therefore 1 \pm 3 = 2\beta$

$\therefore 2\beta - 4 \Rightarrow \beta = 2$

or

$2\beta = -2$ i.e. $\beta = 1$.

(iii) $\vec{a} \times (\vec{b} \times \vec{c}) = 1/2\, \vec{b}$

$(\vec{a}.\ \vec{c})\vec{b} - (\vec{a}\ .\ \vec{b})\vec{c} = 1/2\, \vec{b}$

$\left((\vec{a}\ .\ \vec{c}) - \frac{1}{2}\right)\vec{b} - (\vec{a}\ .\ \vec{c})\ \vec{c} = 1/2\, \vec{b}$

As $\vec{a}$ & $\vec{c}$ are non collinear

$\Rightarrow \vec{a}.\vec{c} \ -1/2 = 0$ & $\vec{a}\ .\ \vec{b} = 0$

$\vec{a}\ .\ \vec{c} = = 1/2$ & $\vec{a} \perp \vec{b} = \alpha = 90^\circ = \pi/2$

$|\vec{a}||\vec{c}| \cos\theta = 1/2$ & $\alpha = \pi/2$

$\Rightarrow \theta = 60^\circ = \pi/3$ & $\alpha = \pi/2$

$\therefore |\alpha - \theta| = |\pi|2 - \pi|3| = 30^\circ$

9. (a)

RHL : $\lim\limits_{h\to 0} \frac{\tan(\pi \sin^2(h)) + (h+0)^2}{h^2}$

i.e $\lim\limits_{h\to 0}\left(\frac{\tan(\pi/\sin^2(h)) \times \sin^2 h}{\pi \sin^2 h \times h^2}\right) + \lim\limits_{h\to 0}(1)$

ie. $\left(\lim_{h\to 0}\frac{\tan\left(\pi\sin^2 h\right)}{\pi\sin^2 h}\right)\pi\lim\left(\frac{\sin h}{h}\right)^2+1$

ie $\pi + 1$

LHL : $\lim_{h\to 0}\frac{\tan\left(\sin^2(-h)\right)+\left(|-h|-\sin(-h[-h])\right)^2}{(-h)^2}$

(As h → 0 & h > 0 & – h < 0 ∴ [–h] = –1)

$= \lim_{h\to 0}\left(\frac{\tan\left(\pi\sin^2 h\right)}{h}+\left(\frac{h-\sin h}{h}\right)^2\right)$

$(1 \times \pi) + (1 + 1 - 2) = \pi$

$\therefore$ LHL $\neq$ RHL

10. (a)

(i) : $x \to 0$

$(1 + x)^n = 1\ nx$

$\sqrt{1+y^4} = (1+y^4)^{1/2}$

$= 1 + \frac{1}{2}y^4$

$\therefore \lim_{y\to 0}\frac{\sqrt{1+1+\frac{y^4}{2}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2+\frac{y^4}{2}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2}\left(\left(1+\frac{y^4}{4}\right)^{1/2}-1\right)}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2}\left(1+\frac{1}{2}y\frac{4}{4}-1\right)}{y^4} = \frac{1}{4\sqrt{2}}$