CRASH COURSE

JEE ADVANCED 2021-22

PHYSICS

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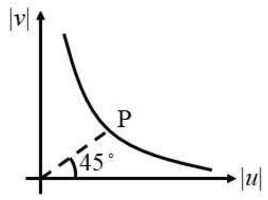
SET-1

(Q1 & 2) Only One Option Correct

1. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular if
   (a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and $|\vec{A}| = |\vec{B}|$
   (b) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
   (c) $\vec{A}$ and $\vec{B}$ are perpendicular but their magnitudes are arbitrary
   (d) $|\vec{A}| = |\vec{B}|$ and their directions are arbitrary

2. The $|u|$, $|v|$ graph for a concave mirror is as shown in figure. Here $|u| > |f|$. A line passing through origin of slope 1 cuts the graph at point P. Then co-ordinates of point P are



(a) $(|2f|, |2f|)$
(b) $(|2f|, |f|)$
(c) $(|f|, |2f|)$
(d) $(|f|, |f|)$

(Q3) More Than One Option Correct

3. Units off CR^2 is/are (C = capacitance and R = resistance).

(a) henry
(b) $\dfrac{\text{volt} - \text{second}}{\text{ampere}}$
(c) $\dfrac{\text{volt}}{\text{ampere}}$
(d) $\dfrac{\text{joule}}{\text{ampere}^2}$

(Q4) Matrix Match

4. For component of a vector $\vec{A} = (3\hat{i} + 4\hat{j} - 5\hat{k})$, match the following table

Table-1	Table-2
(a) Along y-axis	(p) 5 unit
(b) Along another vector $(2\hat{i} + \hat{j} + 2\hat{k})$	(q) 4 unit
(c) Along $(6\hat{i} + 8\hat{j} + 10\hat{k})$	(r) Zero
(d) Along another vector $(-3\hat{i} + 4\hat{j} + 5\hat{k})$	(s) None

(Q5 & 6) Only One Option Correct

5. A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\dfrac{dx}{dt} = c = \dfrac{dy}{dt}$ where a, b and c are constants. The acceleration of the particle is

(a) $(6ac^2x + 2bc^2)\hat{k}$
(b) $(2ax^2 + 6by^2)\hat{k}$
(c) $(4bc^2x + 3ac^2)\hat{k}$
(d) $(bc^2x + 2by)\hat{k}$

6. A particle is dropped from point A at a certain height from ground. It falls freely and passes through three points B, C and D with $BC = CD$. The time taken by the particle to move form B to C is 2 s and from C to D is 1 s. The time taken to move from A to B is

(a) 0.5 s
(b) 1.5 s
(c) 0.75 s
(d) 0.25 s

SET-2

(Q1 & 2) Only One Option Correct

1. The distance between two moving particles at any time is a. If v be their relative velocity and v_1 and v_2 be the components of v along and perpendicular to a. The time when they are closest to each other are

(a) $\frac{av_1}{v^2}$

(b) $\frac{av_2}{v^2}$

(c) $\frac{av}{v_1^2}$

(d) $\frac{av}{v_2^2}$

2. In the one-dimensional motion of a particle, the relation between position x and time t is given by $x^2 + 2x = t$ (here $x > 0$). Choose the correct statement

(a) the retardation of the particle $\frac{1}{4(x+1)^3}$

(b) the uniform velocity of the particle is $\frac{1}{(x+1)^3}$

(c) Both are correct

(d) Both are wrong

(Q3) More Than One Option Correct

3. Let $\vec{v}$ and $\vec{a}$ be the instantaneous velocity and acceleration of a particle moving in a plane. The, rate of change of speed $\frac{dv}{dt}$ of the particle is equal to

(a) $|\vec{a}|$

(b) $\frac{\vec{v}.\vec{a}}{v}$

(c) the component of $\vec{a}$ parallel to $\vec{v}$

(d) the component of $\vec{a}$ perpendicular to $\vec{v}$

(Q4 & 5) Comprehension Type

Passage

At time $t = 0$, particle A is at $(1m, 2m)$ and B is at $(5m, 5m)$. Velocity of B is $\left(2\hat{i} + 4\hat{j}\right)$ m/s velocity of particle A is $\sqrt{2}\,v$ at 45° with x-axis. A collides with B.

4. Value of v is...............m/s.

(a) 5

(b) 15

(c) 25

(d) 10

5. Time when A will collide with B is.............. second.

(a) 0.5 s
(b) 1.5 s
(c) 4 s
(d) 3 s

Answer & Solutions

SET-1

(Q1 & 2) Only One Option Correct

1. (d)

Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$ given

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

$|\vec{A}| = |\vec{B}|$

$\vec{A} \times \vec{B} \perp$ to plane formed by $\vec{A}$ and $\vec{B}$ or $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$`

2. (a)

When object at centre of curvature, image coincides with object.

(Q3) More Than One Option Correct

3. (a, b, d)

Time constant in C-R and L-R circuits are CR and $\frac{\text{L}}{\text{R}}$

CR $= \frac{\text{L}}{\text{R}}$ or $\text{CR}^2 \equiv \text{L}$ units of CR^2 and L are same

$|E| = L\left(\frac{dI}{dt}\right)$ and $U = \frac{1}{2}Li^2$

$\Rightarrow$ Units of L or CR^2, $\frac{V - second}{A}$ and $\frac{J}{A^2}$

(Q4) Matrix Match

4. (a) $\rightarrow$ (q)
(b) $\rightarrow$ (r)
(c) $\rightarrow$ (s)
(d) $\rightarrow$ (s)

(Q5 & 6) Only One Option Correct

5. (a)

$$\frac{d\alpha}{dt} = \frac{dy}{dt} = c$$

$$\frac{d^2\alpha}{dt^2} = \frac{d^2y}{dt^2} = 0$$

$Z = ax^3 + by^2$

$= 3acx^2 + 2bcy$

$$\frac{d^2Z}{dt^2} = 6ac \times \left(\frac{dx}{dt}\right) + 2bc\left(\frac{dy}{dt}\right)$$

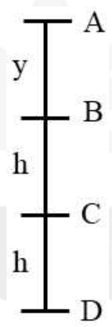
$= 6ac^2 + 2bc^2$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$\left(6ac^2x + 2bc^2\right)\hat{k}$$

6. (a)

$t_{AB} = t$



$y = \frac{1}{2}gt^2$

$y + h = \frac{1}{2}g(t + 2)^2$

$y + 2h = \frac{1}{2}g(t + 3)^2$ h

$\Rightarrow t = .5$ s

SET-2

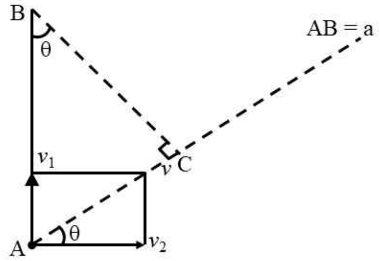
(Q1 & 2) Only One Option Correct

1. (a)

$$V^2 = V_1^2 + V_2^2 \Rightarrow \tan\theta = \frac{V_1}{V_2}$$

$$\cos\theta = \frac{V_2}{\sqrt{v_1^2 + v_2^2}}$$

$= \frac{V_2}{v}$



$\sin\theta = \frac{V_1}{\sqrt{V_1^2 + V_2^2}} = \frac{V_1}{V}$

Time $\frac{AC}{V} = \frac{AB\sin\theta}{V} = \frac{av_1}{v^2}$

2. (a)

$\frac{dt}{dx} = 2(x+1) \Rightarrow v = \frac{dx}{dt}$

$\Rightarrow v = \frac{1}{2(x+1)}$ and $a = \frac{dv}{dt}$

$\Rightarrow -\frac{1}{2(x+1)^2}.\frac{dx}{dt} = -\frac{1}{4(x+1)^3}$

(Q3) More Than One Option Correct

3. (b, c)

Speed $v^2 = v_x^2 + v_y^2$

$\Rightarrow 2V\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{v_x ax + v_y ay}{v} = \frac{\vec{v}.\vec{a}}{v}$

Compound of $\vec{a} \parallel$ to v

(Q4 & 5) Comprehension Type

4. (d)

$\vec{V}_A = V\hat{i} + V\hat{j}$

$\vec{V}_B = 2\hat{i} + 4\hat{j}$

$\vec{V}_{AB} = (V-2)\hat{i} + (V-4)\hat{j}$

$\overrightarrow{AB} = (4\hat{i} + 3\hat{j})$

$\vec{V}_{AB} \uparrow\uparrow \overrightarrow{AB}\, i + \frac{V-2}{4} = \frac{V-4}{3}$

$\Rightarrow$ V = 10

5.(d)

$\left|\overrightarrow{V_{AB}}\right| = 10$

$\left|\overrightarrow{AB}\right| = 5 \Rightarrow t = \dfrac{\left|\overrightarrow{AB}\right|}{\left|\overrightarrow{V_{AB}}\right|} = .5\,\text{sec}$

CHEMISTRY

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[One Option Correct]

1. 1.020g of metallic oxide contains 0.540g of the metal. Calculate the equivalent mass of the metal and hence its atomic mass with the help of Dulong and Petit's law. Taking the symbol for the metal as M. find the molecular formula of the oxide. The specific heat of the metal is 0.216 cal deg^{-1} g^{-1}.
   (a) M_2O_3
   (b) M_4O_3
   (c) M_2O_4
   (d) M_3O_5

2. A partially dried clay mineral contains 8% water. The original sample contained 12% water and 45% silica. The % of silica in the partially dried sample is nearly.
   (a) 50%
   (b) 49%
   (c) 55%
   (d) 47%

3. A mixture in which the mole ratio of H_2 and O_2 is 2:1 is used to prepare water by the reaction,
   $$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$
   The total pressure in the container is 0.8 atm at 20°C before the reaction. Determine the final pressure at 120°C after reaction assuming 80% yield of water.
   (a) 0.8054 atm
   (b) 0.7864 atm
   (c) 0.9744 atm
   (d) 0.6964 atm

4. A mixture of HCOOH and $H_2C_2O_4$ is heated with concentrated H_2SO_4. The gas produced is collected and on treating with KOH solution, the volume of the gas decreases by 1/6th. Calculate the molar ratio of the two acids in the original mixture.
   (a) 2:3
   (b) 6:5
   (c) 4:1
   (d) 8:6

[Integer Type Questions]

5. A plant virus is found to consist of uniform cylindrical particles of 150Å in diameter and 5000Å long. The specific volume of the virus is 0.75 cm^3/g. If the virus is considered to be a single particle, find its molecular mass.
6. On dissolving 2.0g of metal in sulphuric acid, 4.51g of the metal sulphate was formed. The specific heat of the metal is 0.057 cal g^{-1}. What is the valency of the metal and exact atomic mass ?

[Matrix Matching]

7. Match the Column-X and Column-Y:

Column-X	**Column-Y**
(a) 1.6g CH_4	(i) 0.1 mol
(b) 1.7g NH_3	(ii) 6.023×10^{23} electrons
(c) HCHO	(iii) 40% carbon
(d) $C_6H_{12}O_6$	(iv) Vapour density = 15

[One Option Correct]

8. The ratio of the frequency corresponding to the third line in Lyman series of hydrogen atomic spectrum to that of the first line in Balmer series of Li^{2+} spectrum is

   (a) $\frac{4}{5}$ (c) $\frac{4}{3}$
   (b) $\frac{5}{4}$ (d) $\frac{3}{4}$

9. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. Half lives of X and Y are 40000 years and 20 years respectively. In certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20}, the number Y nuclei present in it is:

   (a) 2×10^{17} (c) 4×10^{23}
   (b) 2×10^{20} (d) 4×10^{20}

10. Three isotopes of an element have mass numbers M, $(M + 1)$ and $(M + 2)$. If the mean mass number is $(M + 0.5)$, then which of the following ratios may be accepted for M, $(M + 1)$, $(M + 2)$ in that order?

   (a) 1 : 1 : 1
   (b) 4 : 1 : 1
   (c) 3 : 2 : 1
   (d) 2 : 1 : 1

Answer & Solutions

1. (a)

Mass of oxygen is the oxide = (1.020 − 0.540) = 0.480 gm

Equivalent mass of the metal = $\frac{0.540}{0.480} \times 8 = 9\,\text{gm}$

According to Dulong and Petit's law

Approx. atoms mass = $\frac{6.4}{\text{SP. heat}} = \frac{64}{0.216} = 29.63$

Valency of the metal = $\frac{\text{At. mass}}{\text{Eq. heat}} = \frac{29.63}{9} \approx 3$

Hence,

the formula of the oxide = M_2O_3

2. (d)

	Clay	Silica	Water
Initial stage	43%	45%	12%
Final stage	(92−x)	x	8%

Ratio of silica and clay will remain constant, before and after drying.

$$\frac{45}{43} = \frac{x}{92-x}$$

∴ x = 47%

(e)

3. (b)

$$pH_2 = \frac{2}{3} \times 0.8 = 0.533\ atm$$

$$pO_2 = \frac{1}{3} \times 0.8 = 0.266\ atm$$

$2H_2 + O_2 \rightarrow 2H_2O$

t = 0 0.533 0.266 0

After the reaction = $\frac{0.533 \times 20}{100} = 0.1066,\ \frac{0.266 \times 20}{100} = 0.0533,\ \frac{0.533 \times 80}{100} 0.4264.$

Total pressure = 0.1066 + 0.0533 + 0.4264 = 0.5863

Using Gay-Lussac's law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{0.5863}{293} = \frac{P_2}{393}$$

⇒ P_2 = 0.7864 atm

4. (c)

$$\underset{1\ \text{mol}}{HCOOH} \xrightarrow[\text{Heat}]{\text{conc. } H_2SO_4} \underset{1\ \text{mol}}{CO} + H_2O$$

$$\underset{1\ \text{mol}}{H_2C_2O_4} \xrightarrow[\text{Heat}]{\text{conc. } H_2SO_4} \underset{1}{CO} + \underset{1}{CO_2} + H_2O$$

Let "a" moles of HCOOH and "b" moles of $H_2C_2O_4$ be present in the original mixture

moles of CO formed = $a + b$

moles of CO_2 formed = b

Total moles of gases = $a + 2b$

Now

$$\frac{a+2b}{6} = b$$

$\Rightarrow$ $a = 4b \Rightarrow \frac{a}{b} = 4$

$\Rightarrow$ $a : b = 4 : 1$

5. 114.72

Equivalent mass of SO_4^{2-} radical = $\frac{\text{Ionic mass}}{\text{Valency}}$

$= \frac{96}{2} = 48$

Mass of metal sulphate = 4.51 gm

Mass of metal = 2.0 gm

Mass of sulphate radical = 4.51 − 2 = 2.51 gm

2.51 gm of sulphate combine with 2 gm of metal.

So, 48 gm of sulphate will combine with = $\frac{2}{2.51} \times 48 = 38.24$ gm metal

$\therefore$ Equivalent mass of metal = 38.24 gm

According to Dulong and Petit's law

Approximate atomic mass = $\frac{6.4}{\text{Specific heat}} = \frac{6.4}{0.057} = 112.5$

Valency = $\frac{\text{Approximate atomic mass}}{\text{Equivalent mass}} = \frac{112.5}{38.24} \approx 3$

Exact atomic mass = 38.24 × 3 = **114.72**

6. (a) → (i), (ii),

(b) → (i), (ii),

(c) → (iii), (iv),

(d) → (iii)

(a) 1.6g CH_4 = $\frac{1.6}{16} = 0.1$ mole

$= 0.1 \times 6.022 \times 10^{23} \times 10$

$= 6.022 \times 10^{23}$ electron.

(b) 1.7g NH_3 = $\frac{1.7}{17} = 0.1$ mole

= 0.1 × 6.022 × 10^{23} × 10

= 6.022 × 10^{23} electron.

(c) % of "c" = $\frac{12}{30} \times 100 = 40\%$

MM = 2 × VD

VD = $\frac{MM}{2} = \frac{30}{2} = 15$

(d) % of "c" = $\frac{6 \times 12}{180} \times 100 = \frac{72}{180} \times 100 = 40\%$

7. (d)

For third line in Lyman series.

$n_1 = 1$; $n_2 = 4$

$$V_H = \frac{C}{\lambda} = C \cdot R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= C.\ (R_H)\ (1)^2 \left[\frac{1}{1^2} - \frac{1}{4^2}\right]$$

$$V_H = \frac{15}{16} R_H C$$

For first line in Balmer series for Li^{2+}

$n_1 = 2$; $n_2 = 3$

$$V_{Li^{2+}} = (R_H)(Z^2)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= (R_H)(3^2)\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= C\ R_H \times 9 \times \frac{5}{36} = \frac{5}{4} CR_H$$

$$\therefore \quad \frac{V_H}{V_{Li^{2+}}} = \frac{15}{6} \times \frac{4}{5} = \frac{3}{4}$$

8. (a)

$$X \xrightarrow{\lambda_x} Y \xrightarrow{\lambda_y} Z$$

At equilibrium

$\lambda_x N_x = \lambda_y N_y$

$$N_y = \frac{\lambda_x}{\lambda_y} \times N_x$$

$$= \frac{(t_{1/2})_y}{(t_{1/2})_x} \times N_x$$

$$= \frac{20}{40000} \times 4 \times 10^{20}$$

$= 2 \times 10^{17}$

9. (b)

Let the ratio is, M : (M + 1) : (M + 2) = x : y : z

Mean atomic mass = $\dfrac{M\times x+(M+1)\times y+(M+2)\times z}{(x+y+z)}$

$$M + 0.5 = \frac{xM+y(M+1)+z(M+2)}{(4+1+1)}$$

$$= \frac{4M+1(M+1)+1(M+2)}{6}$$

$$= \frac{6M+3}{6} = \frac{3(2M+1)}{6}$$

$$= M+\frac{1}{2} = M+0.05 = RHS$$

Hence, "b" is the correct option

10. (a) (c)

$$mvr = \frac{nh}{2\pi}$$

$$E_n = E_1 \times \frac{z^2}{n^2}$$

MATHEMATICS

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1. If z, z_2 are non-zero complex numbers such that $|z_1| = |z_2| = |z_1 + z_2|$ then z_1/z_2 can be
   (a) 1
   (b) ω
   (c) ω^2
   (d) -1

2. Modulus of complex number whose reciprocal is
   Match the entries in Column I with entries in Column II

Column-I	Column-II
(a) $\dfrac{1}{a} + \dfrac{1}{b+ic}$	(p) $\dfrac{\sqrt{a^2+b^2}\ \sqrt{a^2+c^2}}{\lvert b-c\rvert}$
(b) $\dfrac{1}{a-ib} - \dfrac{1}{a-ic}$	(q) $\sqrt{a^2+(b+c)^2}$
(c) $\dfrac{b}{a+ib} + \dfrac{c}{a-ic}$	(r) $\dfrac{\lvert a\rvert\ \sqrt{b^2+c^2}}{\sqrt{(a+b)^2+c^2}}$
(d) $\dfrac{1}{a+ib+ic}$	(s) $\dfrac{\sqrt{a^2+b^2}\ \sqrt{a^2+c^2}}{\lvert a\rvert\ \lvert b+c\rvert}$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

3. Let α, β be roots of the equation
$ax^2 + bx + c = 0$, then equation whose roots are
Match the entries in Column-I with entries in Column-II

Column-I	Column-II
(a) $-1/\alpha, -1/\beta$	(p) $ax^2 + 2bx + 4c = 0$
(b) $-\alpha, -\beta$	(q) $a^2x^2 + (2ac - b^2) + c^2 = 0$
(c) α^2, β^2	(r) $cx^2 - bx + a = 0$
(d) $2\alpha, 2\beta$	(s) $ax^2 - bx + c = 0$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

4. **Statement-I :** If all the four roots of
$x^4 - 4x^3 + ax^2 - bx + 1 = 0$ are positive,
than $a = 6$ and $b = 4$.
Statement-II : If polynomial equation $P(x) = 0$ has four positive roots, then the polynomial equation $P'(x) = 0$ has atleast 3 positive roots.
(a) Statement-I is True, Statement-II is True; Statement-II is correct explanation for Statement-I.
(b) Statement-I is True, Statement-II is true; Statement-II is not a correct explanation for Statement-I.
(c) Statement-I is True, Statement-II is False.
(d) Statement-I is False, Statement-II is True.

5. Let $a, b, c \in \mathbf{C}$ such that $a + b + c = 0$.
If $|a| = |b| = |c| = 1$, then $|a - b|^3 + |b - c|^3 + |c - a|^3 - 3\,|a - b|\,|b - c|\,|c - a|$ is equal to

6. $a, b, c \in \mathbf{R}$ and a, b, c are in A.P. Match the expression in Column-I with the conditions/properties in Column-II.

Column-I	Column-II
(a) a^2, b^2, c^2 are in A.P.	(p) $a = b = c$
(b) a^2, b^2, c^2 are in G.P.	(q) $-\frac{1}{2}a, b, c$ are in G.P.
(c) a^2, b^2, c^2 are in H.P.	(r) $a, b, -\frac{1}{2}c$ are in G.P.
(d) $a + b + c = \frac{3}{2}$	(s) $b = \frac{1}{2}$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

7. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement-I : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

Statement-II : The number b_1, b_2, b_3, b_4 are in H.P.

(a) Statement-I is false and Statement-II is true.
(b) Statement-I is true and Statement-II is false
(c) Statement-I and Statement-II both are true
(d) Statement-I and Statement-II both are false

8. **Statement-I :** $\dfrac{1^2}{(1)(3)} + \dfrac{2^2}{(3)(5)} + \ldots + \dfrac{n^2}{(2n-1)(2n+1)} = \dfrac{n(n+1)}{2(2n+1)}$

Statement-II : $\dfrac{1}{(1)(3)} + \dfrac{2}{(3)(5)} + \ldots + \dfrac{1}{(2n-1)(2n+1)} = \dfrac{1}{2n+1}$

(a) Statement-I is false and Statement-II is true.
(b) Statement-I is true and Statement-II is false
(c) Statement-I and Statement-II both are true
(d) Statement-I and Statement-II both are false

Paragraph Question

9. Given a sequence $t_1, t_2, \ldots$ if its possible to find a function $f(r)$ such that

$$t_r = f(r + 1) - f(r)$$

then

$$\sum_{r=1}^{n} t_r = f(n + 1) - f(1)$$

(i) Sum of the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{4r^2 - 1}$ is

(a) 2
(b) 1
(c) 1/2
(d) 1/4

(ii) If $u_1 = 1$, $u_{n+1} = 2u_n + 1$, then u_{n+1} equals
(a) $2^n + 1$
(b) $2^{n+1} - 1$
(c) $2^n - 2$
(d) $2^{n+1} - 2$

(iii) If $x_n = 1^2 + (2)(2^2) + 3^2 + (2)(4^2) + \ldots$

$= n(n + 1)^2/2$ if n is even, then $\dfrac{x_{51}}{(13)(51^2)}$ is

10. Let m and n be two positive integers such that $m \geq n$. The number of ways of Match the entries in Column I with entries in Column II

	Column-I		**Column-II**
(a)	distributing m distinct books among n children	(p)	0
(b)	arranging n distinct books at m places	(q)	m^n
(c)	selecting m persons out of n persons so that two particular persons are not selected	(r)	n^m
(d)	number of functions from $\{1, 2, 3, ... n\}$ to $\{1, 2, 3, ...m\}$	(s)	$(^mC_n)\ (n!)$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	p	r	s

Answer & Solutions

1. (b),(c)

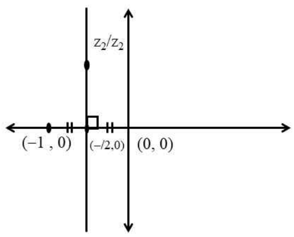
$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}. \quad \text{But } |z_1| = |z_2| = 1$$

$\therefore \quad \left|\frac{z_1}{z_2}\right| = 1$

Also, $\dfrac{|z_1 + z_2|}{|z_2|} = \left|\dfrac{z_1}{z_2} + 1\right| = 1$

$\therefore \quad \left|\dfrac{z_1}{z_2} - (0+0i)\right| = \left|\dfrac{z_1}{z_2} - (-1-0i)\right| = 1$

$\therefore \quad z_1/z_2$ lies on $\perp$ bisector of line segment joining $0 + 0i$ & $(-1 + 0i)$



$\therefore \quad$ Re $(z_1/z_2) = -1/2$

$\therefore \quad z_1/z_2 = -1/2 + ai$

But , $\left|\dfrac{z_1}{z_2}\right| = 1$

$\therefore \quad |-1/2 + ai| = 1$

$\Rightarrow \quad \dfrac{1}{4} + a^2 = 1 \Rightarrow a^2 = 3/4 \Rightarrow a = \pm\sqrt{3}/2$

$\therefore \quad \dfrac{z_1}{z_2} = -1/2 \pm \dfrac{\sqrt{3}}{2} 1$

$= ώ, ώ^2$

2. (a) $\rightarrow$ (r),(b) $\rightarrow$ (p),(c) $\rightarrow$ (s),(d) $\rightarrow$ (q)

(a)

$$\left|\frac{a + b + i\,c}{a\,(b + i\,c)}\right| = |1/z|$$

$\Rightarrow \quad \dfrac{1}{|z|} = \dfrac{\sqrt{(a + b)^2 + c^2}}{|a|\,\sqrt{b^2 + c^2}}$

$\therefore \quad |z| = \dfrac{|a|\,\sqrt{b^2 + c^2}}{\sqrt{(a + b)^2 + c^2}}$

$a \rightarrow r$

(b) $\left|\frac{1}{z}\right| = \left|\frac{a - ic - a + ib}{(a - ib)(a - ic)}\right|$

$\frac{1}{|z|} = \frac{|i(b - c)|}{|a - ib|\ |a - ic|} = \frac{|b - c|}{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}$

$\therefore \quad |z| = \frac{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}{|b - c|}$

$b \to p$

(c) $\left|\frac{1}{z}\right| = \left|\frac{ab - ib + ac + ibc}{(a + ib)(a - ib)}\right|$

$= \frac{|a(b + c)|}{|(a + ib)(a - ic)|}$

$\frac{1}{|z|} = \frac{|a|\ |b + c|}{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}$

$|z| = \frac{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}{|a|\ |b + c|}$

$c \to s$

(d) $\left|\frac{1}{z}\right| = \left|\frac{1}{a + i(b + c)}\right|$

$\frac{1}{|z|} = \frac{1}{\sqrt{a^2 + i(b + c)^2}}$

$\therefore \quad |z| = \sqrt{a^2 + (b + c)^2}$

$d \to q$

3. (a) $\to$ (r),(b) $\to$ (s),(c) $\to$ (q),(d) $\to$ (p)

(a) Replace x by $-1/x$ in $ax^2 + bx + c = o$ to get $a - bx + cx^2 = 0$.

a $\to$ r

(b) Replace x by $-x$ in $ax^2 + bx + c = o$ to get $ax^2 - bx + c^2 = 0$

b $\to$ s

(c) Replace x by $\sqrt{x}$ in $ax^2 + bx + c = o$ to get $a^2 x^2 + (2ac - b^2)\ x + c^2 = 0$

c $\to$ q

(d) Replace x by $x/2$ in $ax^2 + bx + c = o$ to get $ax^2 + 2bx + 4c = 0$

d $\to$ q

4. (b)

let x_1, x_2, x_3, x_4 be the 4 roots of $x^4 - 4x^3 + ax^2 - bx + 1 = O$

$\therefore \quad x_1 + x_2 + x_3 + x_4 = 4$

$x_1\ x_2\ x_3\ x_4 = 1$

$\therefore \quad \underbrace{\tfrac{1}{4}(x_1 + x_2 + x_3 + x_4)}_{\text{A.M. of } x_1, x_2, x_3, x_4 = 1} \qquad \underbrace{(x_1\ x_2\ x_3\ x_4)^{1/4}}_{\text{G.M. of } x_1, x_2, x_3, x_4 = 1}$

$\Rightarrow \quad x_1 = x_2 = x_3 = x_4 = 1.$

$\therefore$ $x^4 - 4x^3 + bx^2 - bx + 1 = (x-1)^4$

$\Rightarrow$ a = 6 & b = 4

Also, Between any 2 roots of P(x) lies one root of P'(x) where P(x) is a polynomial

$\therefore$ Statement 1 & 2 both are true (b)

5. The integer 0

$$|b-c|^2 + |b+c|^2 = 2\left(|b|^2 + |c|^2\right)$$

$$|b-c|^2 + |-a|^2 = 2(1+1) = 1$$

$$\therefore \quad |b-c|^2 = 3$$

$$|b-c| = \sqrt{3} = |a-b| - |a-c|$$

$$\therefore \quad |a-b|^3 + |b-c|^3 + |c-a|^3 - 3|a-b|\,|b-c|\,|c-a|$$

$$3\sqrt{3} + 3\sqrt{3} + 3\sqrt{3} - 3\sqrt{3}\ \sqrt{3}\ \sqrt{3} = 0.$$

6. (a) $\rightarrow$ (p),(b) $\rightarrow$ (p),(c) $\rightarrow$ (p,q,r),(d) $\rightarrow$ (s)

$2b = a + c$

& $2b^2 = a^2 + c^2$

(a) $(a+c)^2 = (2b)^2$

$= 4b^2$

$= 2(2b^2)$

$a^2 + c^2 + 2ac = 2a^2 + 2c^2 \quad (a-c)^2 = O$

$\Rightarrow$ a = c but 2b = a + c

$\Rightarrow$ a = b = c

$a \rightarrow p$

(b) $(b^2)^2 = a^2c^2$

$b^2 = \pm\, ac$

$\therefore$ $b^2 = ac$

$\Rightarrow$ a,b,c are in GP

Already, a,b,c are in ap

$\therefore$ a = b = c

$\therefore$ $b \rightarrow p.$

(c) $b^2 = \dfrac{2a^2c^2}{a^2+c^2}$

but, $b^2 = \left(\dfrac{a+c}{2}\right)^2$

$$\left(\frac{a+c}{4}\right)^2 = \frac{2a^2c^2}{a^2+c^2}$$

$$\underbrace{\left((a+c)^2 + 2ac\right)}_{\downarrow} \qquad \left((a-c)^2\right) = 0 \longrightarrow a = b = c$$

$\frac{-a}{2}$, b,c are in GP
or
a,b, − c/2 are in GP

c → p, q, r.

(d) $a + b + c = 3/2$

$b + 2b = 3/2$

$3b = 3/2$

$\therefore$ $b = 1/2$

$\therefore$ d → s

7. (b)

Let a_1, a_2, a_3, a_4, be in GP

$\therefore$ $a_1 = a,\ a_2 = ar,\ a_3 = ar^2,\ a^4 = ar^3$ with r = common ratio.

$\therefore$ $b_1 = a$

$b_2 = a + ar = a(1 + r)$

$b_3 = a + ar + ar^2$

$= a(1 + \lambda + \lambda^2)$

$b4 = a(1 + \lambda + \lambda^2 + \lambda^3)$

Now,

$b_2 - b_1 \neq b_3 - b_2$

$\therefore$ b_1, b_2, b_3, b_4 are not in AP

Also, $\frac{b_2}{b_1} \neq \frac{b_3}{b_2}$

$\therefore$ b_1, b_2, b_3, b_4 are not in GP.

Also, $1/b_2 - 1/b_1 \neq 1/b_3 - 1/b_2$

$\therefore$ $\frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3}, \frac{1}{b_4}$ are in AP

$\therefore$ b_1, b_2, b_3, b_4 are not in H.P

$\therefore$ Statement (1) is true & (2) statement is false

8. (b)

$$tr = \frac{r^2}{(2r-1)(2r+1)}$$

$$4tr = \frac{4r^2 - 1 + 1}{(2r-1)(2r+1)}$$

$$4tr = 1 + \frac{1}{2}\left(\frac{1}{(2r-1)(2r+1)}\right)$$

$$4\sum_{r=1}^{n} tr = \sum_{r=1}^{n} 1 + \frac{1}{2}\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)}$$

$$4\sum_{r=1}^{n} tr = n + \frac{1}{2}\left(\frac{1}{1} - \frac{2}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots \frac{-1}{2n+1}\right)$$

$$= n + \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$= n + \frac{2}{2n+1} = \frac{n(2n+1)+n}{2n+1}$$

$$\therefore \quad \sum_{r=1}^{n} tr = \frac{1}{4}\left(\frac{2n(n+1)}{2n+1}\right)$$

$$= \frac{2n(n+1)}{2(2n+1)}$$

And, $\quad tr = \dfrac{1}{(2r-1)(2r+1)}$

$$= \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$$

$$\sum_{r=1}^{n} tr = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$= \frac{n}{2n+1}$$

$\therefore$ Statement (1) is true , Statement (2) is false.

9. (1) $\rightarrow$ (c),(2) $\rightarrow$ (b),(3) $\rightarrow$ (2)

(1)

$$\sum_{r=1}^{\infty} \frac{1}{4r^2 - 1} = \sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(\sum_{r=1}^{n}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)\right)$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = 1/2 \quad \text{(C)}$$

(2)

$$U_{n+1} - Un = 2Un + 1 - 2U_{n-1} - 1$$

$$= 2Un - 2U_{n-1}$$

$$= 2\ (Un - U_{n-1})$$

$$= 2.2(Un - 1 - Un - 2)$$

$$= 2.2\ (U_{n-1} - U_{n-2}) \ldots.$$

$$= 2^{n-1}\ (U_2 - U_1) = 2^{n-1}\ (3-1)$$

$$= 2^n$$

$$\therefore \quad U_{n+1} = 2^n + U_n = 2^n + 2^{n-1} + U_{n-1}$$

$$= 2^n + 2^{n-1} + 2^{n-2} + U_{n-2}$$

$$= 2^n + 2^{n-1} + \ldots. + 2^1 + U_1 = 2\left(\frac{2^n - 1}{2-1}\right) + 1 = 2^{n+1} - 1$$

(3)

$$x_{51} = x_{50} + 51^2$$

$$= \frac{25(51)^2}{2} + 51^2$$

$= 25\ (51)^2 + 51$
$= 51^2 \times 26 = 51^2 \times 13 \times 2$

$$\frac{x_{51}}{13 \times 51^2} = 2$$

10. (a) → (r),(b) → (s),(c) → (p),(d) → (q)
(1) m disticut books can be distributed among n children = n^m ways

(2) ${}^mC_n \times n!$

(3) Out of n persons, 2 are not selected
∴ m persons are to be
selected from n–2 persons. But, $m \geq n > n - 2 \Rightarrow m > n - 2$
∴ Not possible
(4) Each members of domain can be mapped in m ways & domain has n members
∴ No of maps = m^n