

CLASS 11

# CBSE

## PHYSICS

- ▶ Specially designed eBook for complete CBSE syllabus
- ▶ CBSE preparation strategy & direction, speedy revision
- ▶ Chapter-wise important Problem-Solving
- ▶ Theory, Formulas & Diagrams to crack CBSE
- ▶ Updated New Pattern Objective questions
- ▶ Detailed high quality video lectures by experienced faculties
- ▶ CBSE full preparation for Class XI & XII



**misostudy**  
connect smart learning

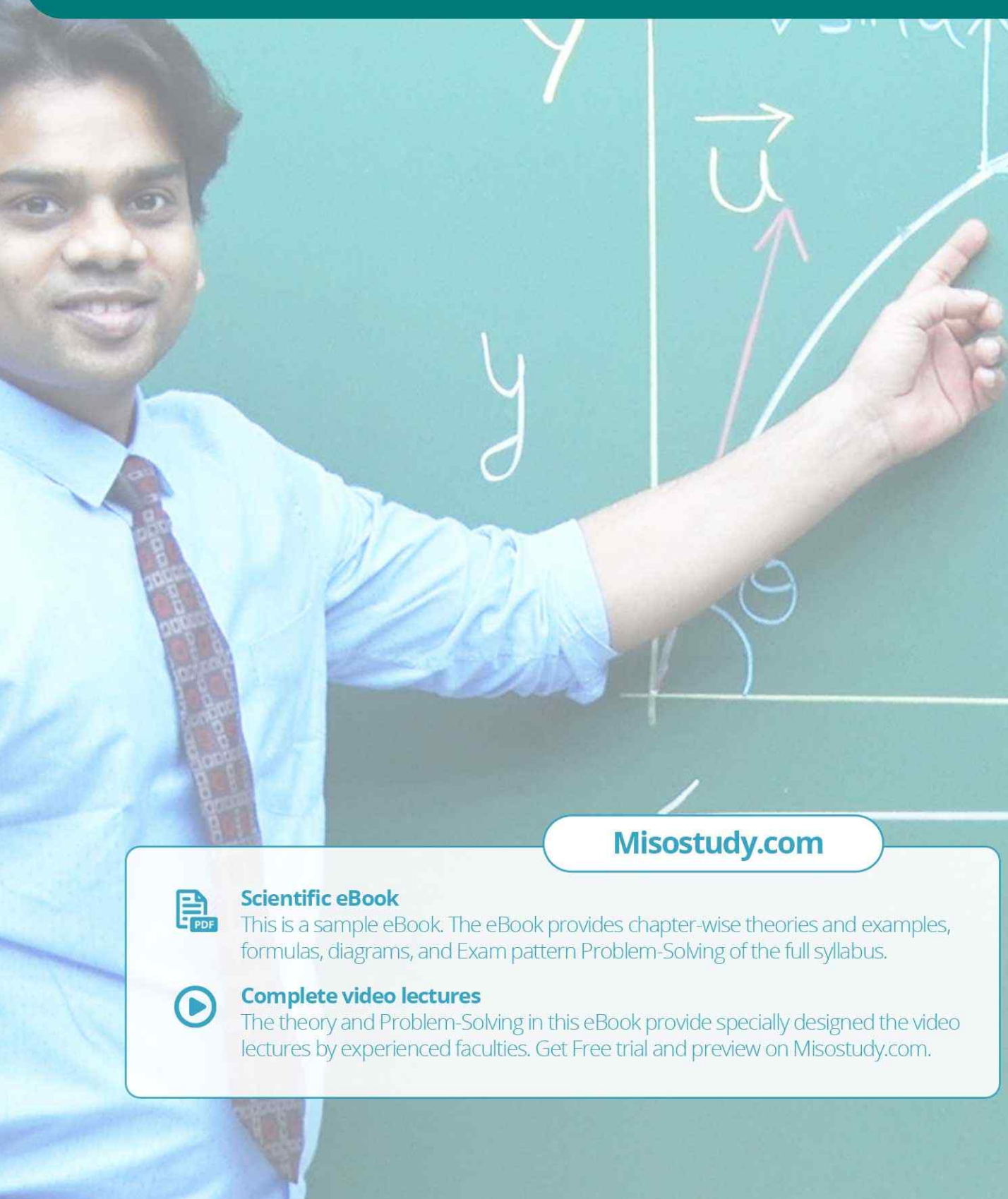
**JEE·NEET·AIIMS·CBSE·FOUNDATION**  
[www.misostudy.com](http://www.misostudy.com)

☎ 8929803804 (MON-FRI: 9am-6pm) [support@misostudy.com](mailto:support@misostudy.com)

MISO STUDY INDIA PVT. LTD.  
2ND FLOOR 65-A, OMPRO TOWER, KALU SARAI, NEW DELHI, DELHI 110016

# Class 11 | Physics

## 01 Units and Measurement



Misostudy.com



### Scientific eBook

This is a sample eBook. The eBook provides chapter-wise theories and examples, formulas, diagrams, and Exam pattern Problem-Solving of the full syllabus.



### Complete video lectures

The theory and Problem-Solving in this eBook provide specially designed the video lectures by experienced faculties. Get Free trial and preview on Misostudy.com.

## 01. Introduction

- Biology; Bios = life, Logos = Study, means study of life is biology.
- “**Biology** is the science of life forms and living processes”
- **Systematic** and **monumental** description of life forms made human to make a detailed system of identification, nomenclature and classification of organisms i.e. **Taxonomy**.
- Hence the study of identification, nomenclature and identification is called taxonomy.
- All the organisms have been evolved by a very long and complex process of **evolution**, so they all are related to each other by sharing of some **common genetic material** but up to varying degrees. This sharing may be less or more among different cases..
- When human came to know this fact then he humbled and led to cultural movements for conservation of **biodiversity**.
- Sharing of common characters was also proved when human studied the similarities among living organisms both **horizontally** and **vertically**.
- The living world is full of amazing diversity of living organisms.
- The diversity of habitats of organisms is also very vast and amazing.
- This diversity makes us deeply reflect on “What indeed is life” ? This question actually asks to solve two problems.
  - (i) First is a technical → What living is as opposed to the non living means **Living v/s Non living**.
  - (ii) Second is a philosophical one → what the **purpose of life** is?  
As scientists we will try to solve the first question, because the second question is more related to philosophy rather science.

## 02. Characters of Living Organism

Following are the main characters of “living”-

GROWTH }  
REPRODUCTION } Not the defining properties/characters/features

METABOLISM }  
CELLULAR ORGANISATION } Defining properties/characters/features  
CONSCIOUSNESS }

The character which has no exception is called as **defining property** of life.

## Growth

- Increase in mass or overall size of a tissue or organism or its parts is called growth.
- Increase in mass and increase in number of individuals are twin characters of growth.
- Growth is an irreversible permanent increase in size of an organ or its parts or even of an individual cell.
- Growth is of two types :
  - (a) **Intrinsic growth** :- This growth is from inside of the body of living organisms.
  - (b) **Extrinsic growth** :- This growth is from outside i.e. accumulation of material on any body surface Non living exhibits this type of growth.
- Growth is of two types :
  - (a) **Indeterminate growth = Unlimited growth** → Growth which occurs continuously throughout their life span is indeterminate growth or unlimited growth. It occurs in plants and not in animals.
  - (b) **Determinate growth = Limited growth** → Growth which occurs only up to a certain age is determinate growth or Limited growth. It occurs in animals. However, cell division occurs in certain tissues to replace lost cells.
- In majority of higher plants and animals, growth and reproduction are mutually **exclusive events**.
- Because both living and nonliving exhibit growth so it can not be taken as defining property.
- Growth from inside (intrinsic growth) can be taken as **defining property**.

## Reproduction

Reproduction is one of the fundamental characteristics of living organisms. It can be defined as the production of new individuals of same kind by the grown up individuals. It is characteristic exhibited by living organisms which can produce new young ones of their own kind. There are two modes of reproduction — **asexual** and **sexual**.

- In lower organisms like yeasts and Hydra, budding takes place in which new individuals are produced by the formation of an outgrowth known as 'bud'.
- Fragmentation is another mode of asexual reproduction, as in this, body of an organism (parent body) breaks up into two or more parts (known as fragments) each of which grows into a new individual. It is also quite common in filamentous algae, fungus, bryophytes (at protonema stage which occurs during life cycle in mosses).
- Planaria (flat worms) exhibit an extraordinary ability to regenerate its lost body parts completely (which is known as true regeneration).

## Metabolism :

- The sum total all the chemical reactions occurring in our body is metabolism.
- All living organisms, both unicellular and multicellular exhibit metabolism.
- No non-living object shows metabolism.
- In this way metabolism is a defining character of living organisms because it has no exceptions.
- Now we have known most of the chemical or metabolic reactions occurring in our body so we can demonstrate many of them in a cell free medium or in a test tube in lab..
- The isolated metabolic reaction outside the body of an organism, performed in a test tube (in-vitro) is neither living nor nonliving.
- These isolated reaction can be regarded as living things, but they are definitely living reactions because they are similar to the reactions performing in our body.

## 01 Units and Measurement

- Here we should not forget the fact that metabolism is the total sum of all the chemical reactions performing in our body, it is not the sum of few or more living reactions.
- All organisms are made of small or big chemicals perform thousands of reactions and form some other chemicals also in the bodies of living organisms.

### Consciousness

- Most obvious and technically complicated feature of all living organism. All living organisms are able to detect changes, i.e., sense their surroundings and can also respond to them. This is known as sensitivity which is defined as the ability to detect changes in the environment and to give response towards it accordingly. Any change that can be detected by an organism is called stimulus. This can be physical (like intensity, duration, direction of light, sound, change in temperature, duration of day length, i.e., photoperiod, etc.), chemical (like acids, pollutants, etc.) or biological (like other organisms).

## 03. Diversity In The Living World

- We consider vast areas like forest, desert, plateau etc. we find that number and kinds of living organisms increase many folds. These different kinds of plants, animals and other organisms are referred to as '**Biodiversity**' of this earth.
- **Biodiversity** is the number and various kinds of organisms found on earth. It stands for the variability found among living organisms inhabiting this world.
- **Biodiversity (Greek word bios = life; diversity = forms) or biological diversity** can be defined as the vast array of species of microorganisms, algae, fungi, plants, animals occurring on the earth either in the terrestrial or aquatic habitats and the ecological complexes of which they are a part.
- According to **IUCN** (International Union of Conservation of Nature and Natural resources), currently known and described species of all organisms are between **1.7-1.8 million**. These millions of plants, animals and other organisms in the world cannot be recognised, studied or described by an individual on its own.
- There is need to standardize the names of all living organisms after proper identification, in order to study such diverse life forms. Therefore, for better understanding of biodiversity scientists have established a definite system of principles, procedures and terms which identifies, categories and assigns specific name to each and every organism known to us. Such systems are acceptable to all biologists all over the world.
- The scientific need for simple, stable and internationally accepted systems for naming the living organisms of the world has generated, a process called "**Nomenclature**". And, before assigning a specific name to an organism, one should determine or know its kind or features correctly, so that one can identify it in each every part of the world. This is known as "**Identification**"

## 04. Taxonomy


- This word was proposed by **A.P. De Candolle** in his book "**Theories elementaire de la botanique**" (Theory of elementary botany)

- **Taxonomy includes study of following 4 points**

<b>Characterization</b>	-	Identification of character
<b>Identification</b>	-	Identification of living organisms
<b>Nomenclature</b>	-	Nomenclature of living organisms
<b>Classification</b>	-	Classification of living organisms in groups

## 05. Systematics


- The term “**Systematics**” was proposed by **Linnaeus**. The word systematics is derived from the latin word “systema” which means systematic arrangement of organisms.
- According to G. Simpson** : Systematics is the study of diversity of organisms and all their comparative and evolutionary relationship.

**NOTE**  It includes description of external morphological characters of plants of living organisms.

eg. Morphological characters of Root, Stem, Leaves, Flowers.

## 06. New Systematics or Neosystematics or Biosystematics

- **Neo-systematics** – A new branch – Name given by **Julian Huxley (1940)**
- It includes description of all the characters (**external + internal**)  
e.g. Anatomical characters, Histological characters, Embryological characters, Identical characters
- It is used to know the **inter-relationship** between living organisms.

**NOTE**  New systematics is mainly based on **evolutionary** as well as **genetic relationship** as compared to morphological characters.

## 07. Types of Taxonomy

- **Alpha taxonomy or classical taxonomy** : It is based on **external morphology** of plants.
- **Beta taxonomy or Explorative taxonomy** : Besides external **morphology** it also includes **internal** characters like embryological, cryological, anatomical characters etc.
- **Omega taxonomy or Encyclopaedic taxonomy** : Omega taxonomy have widest scope. It is based on **all the information** or data available about plants.
- **Chemotaxonomy** : The uses of chemical characters of plants in classification or in solving taxonomic problems is called chemotaxonomy or chemical taxonomy. It is based on the **chemical constituents** of plants. The fragrance and taste vary from species to species. The basic chemical compounds used in chemotaxonomy are **alkaloids, carotenoides, tannins, polysaccharide, nucleic acids, fatty acids, amino acids, aromatic compounds** etc.

- **Cytotaxonomy** : The use of cytological characters of plants in classification or in solving taxonomic problems is called cytotaxonomy. Cytological data have been used in many cases to find out the affinities among genera.
- **Karyotaxonomy** : Based on characters of **nucleus and chromosomes**. Pattern of **chromosomal bands** (dark bands and light bands) is most specific character.
- **Alpha taxonomy or classical taxonomy** : It is based on **external morphology** of plants.
- **Bete taxonomy or Explorative taxonomy** : Besides external **morphology** it also includes **internal** characters like embryological, cryological, anatomical characters etc.
- **Omega taxonomy or Encyclopaedic taxonomy** : Omega taxonomy have widest scope. It is based on **all the information** or data available about plants.
- **Chemotaxonomy** : The uses of chemical characters of plants in classification or in solving taxonomic problems is called chemotaxonomy or chemical taxonomy. It is based on the **chemical constituents** of plants. The fragrance and taste vary from species to species. The basic chemical compounds used in chemotaxonomy are **alkaloids, carotenoides, tannins, polysaccharide, nucleic acids, fatty acids, amino acids, aromatic compounds** etc.
- **Cytotaxonomy** : The use of cytological characters of plants in classification or in solving taxonomic problems is called cytotaxonomy. Cytological data have been used in many cases to find out the affinities among genera.
- **Karyotaxonomy** : Based on characters of **nucleus and chromosomes**. Pattern of **chromosomal bands** (dark bands and light bands) is most specific character.
- **Adansonian system or Phenetic Classification or Numerical Classification.**
  - ❖ Proposed by “**Sokal and Sneath**”
  - ❖ In it plants are classified on the basis of number of similarities and dissimilarities.
  - ❖ In this, importance to any one character is not given, **all characters have same importance.**  
While in **natural** classification **floral characters** have importance than morphological characters.

## 08. Significance of Taxonomy

- Most significant feature is **identification** of living organism.
- With the help of taxonomy **diversity** of living being can be studied easily.
- Maximum diversity of living beings are found in **tropical rain forests**. These forest have heavy rain fall through out the year. In **India** maximum tropical rain forest are found in **Andaman & Nicobar** and in all **eastern states** (Assam, Meghalaya, West Bengal etc.)
- At present, 300 lakh (**30 million**) type of living organism are found on our earth.
- 17 lakh (**1.7 million**) type of living organism have been **discovered till now**.
- Out of them **12 lakh** types are **animal** and **5 lakh** types are **plants**.

## 09. Taxonomic Category

Classification is not a single step process but involves hierarchy of steps in which each step represent a rank or category.

- **Species :**

Taxonomic studies consider a group of individual organism with fundamental similarities as a species. One should be able to distinguish one species from the other closely related species based on the distinct morphological difference.

- **Genus :**

Genus comprises a group of related species which has more characters in common in comparison to species of other genera.

- **Family :**

Family has a group of related genera with still less number of similarities as compared to genus and species. Families are characterized on the basis of both vegetative and reproductive feature of plant species.

For example : Three different genera **Solanum**, **Petunia** and **Datura** are included in family solanaceae.

- **Order :**

Order being a higher category is the assemblage of families which exhibit a few similar character.

For example : Plant families like **convolvulaceae**, **Solanaceae** are included in the order **polymoniales** mainly based on the floral characters.

- **Class :**

Class includes organism of related orders having less similarities than orders.

- **Division :**

Division includes all organisms belonging to different classes having a few common characters. There 7 main taxonomic categories. They are obligate or essential or broad categories i.e. they are strictly used at the time of any plant classification. There are some extra or sub categories, like sub division, sub order, sub family, etc. They are used only when they are needed.

Biggest group →	— Kingdom
	— Division/Phylum
	— Class
	— Order - Cohort
	— Family
	— Genus
	— Species

- ❖ The classification of any plant or animal is written in **descending** or ascending order.
- ❖ Hierarchy – **Descending** or ascending arrangement of taxonomic categories is known as hierarchy.
- ❖ **Species** : Smallest taxonomic category → It is basic unit of classification.



## 10. Nomenclature

- **Polynomical system :**
  - ❖ According to this system name of any plant consists of many words.
  - ❖ For eg. Caryophyllum → Caryophyllum saxatilis folis gramineus umbellatis corymbis
  - ❖ Based of morphology mainly
  
- **Trinomical system :**
  - ❖ Proposed by **Lamarck**.
  - ❖ According to this system name of any plant or species is composed of three names-
    - **Generic name**
    - **Specific epithet**
    - **Subspecific name (Name of variety)**
  - ❖ When members of any species have large variations then trinomial system is used.
  - ❖ On the basis of dissimilarities this species is classified into sub species. eg.
    - Brassica oleracea var. botrytis (Cauliflower)**
    - Brassica oleracea var. capitata (Cabbage)**
    - Brassica oleracea var. caulorapa (Knol-Khol)**
  
- **Binomial system :**
  - ❖ Binomial system was first proposed by **Gaspard Bauhin** in his book -“**Pinax Theatre Botanica**”
  - ❖ **Carolus Linnaeus** : Linnaeus used this nomenclature system for the first time on large scale and proposed scientific name of all the plants and animals.
  - ❖ **Linnaeus** is the **founder** of binomial system.
  - ❖ Linnaeus proposed scientific name of plants in his book “**Species Plantarum**”
  - ❖ It was published on **1 May, 1753**. So this was the initiation of binomial system for plants.
  - ❖ So any name proposed (for plants) before this date is **not** accepted today.
  - ❖ **Linnaeus** proposed scientific name of animals in his book “**Systema Naturae**” (10<sup>th</sup> edition).
  - ❖ This 10<sup>th</sup> edition of **Systema Naturae** was first published on **1 August, 1758**.
  - ❖ So initiation of binomial system **for animals** is believed to be started on **1 August, 1758**.

## 11. ICBN

- **Main rules of ICBN :**
  - ❖ According to binomial system name of any species consists of two components or words.
    - Generic name            Name of genus
    - Specific epithet

e.g.	<i>Solanum tuberosum</i> (Potato)		<i>Mangifera indica</i> (Mango)
	↓	↓	↓
	Generic name	Specific epithet	Generic name      Specific epithet

- ❖ In plant nomenclature (ICBN) tautonyms are not valid i.e. generic name and specific epithet should not be same in plants.  
eg. *Mangifera mangifera*  
But tautonyms are valid in animal nomenclature (ICZN-International Code of Zoological Nomenclature)  
eg. *Naja naja* (Indian cobra), *Rattus rattus* (Rat)
- ❖ First letter of generic name should be in capital letter and first letter of specific epithet should be in small letter.  
eg. *Mangifera indica*
- ❖ But if specific epithet is based on the name of any person, its first letter should be in capital letter.  
eg. *Isoetes pantii*
- ❖ When written with free hand or typed, then generic name and specific epithet should be separately underlined.  
But during printing name should be in italics to indicate their latin origin.
- ❖ Name of scientist (who proposed nomenclature) should be written in short after the specific epithet.  
eg. *Mangifera indica* Linn.
- ❖ Name of scientist should be neither underlined nor in italics, but written in Roman letters (simple alphabets)
- ❖ Scientific names should be derived from Latin (usually) or Greek languages because they are dead languages.
- ❖ Type specimen (Herbarium Sheet) of newly discovered plant should be placed in herbarium (Dry garden).

## 12. Classification

According to “A.P. de Candolle”, Classification is of two types

(i) **Empirical Classification**

(ii) **Rational Classification**

• **Empirical Classification :**

- ❖ In this type, the actual nature or character of plants is not considered.
- ❖ Plants are classified on the basis of their alphabetical order.
- ❖ In this way plants are classified into 26 groups.

• **Rational Classification**

In this classification, plants are classified on the basis of their actual character or nature i.e. by viewing the characters.

### 13. Taxonomical Aids

- Accurate classification and identification of organisms is required which needs field studies and intensive laboratory work. This is done after collection of actual specimens of plants and animal species which is the primary source of all taxonomic studies.
- Hence, these taxonomical studies help in Fundamental study of different living organisms. Also aid in their systematic study. Information gathered is stored with specimens for future studies.

#### Herbarium

- It is defined as “store house of collected plant specimens that are dried, pressed and preserved on sheets”. Further, these sheets are arranged in the sequence of a universally accepted system of classification.
- Such herbaria serve as quick source of reference in taxonomical studies. It also provides information about the local flora as well as flora of distant areas. This information is also useful in locating wild varieties and relatives of economically important plants.

#### List of some Herbaria of the world :

- (i) Royal Botanical Gardens, Kew (England)
- (ii) Central National Herbarium, Calcutta

### 14. Botanical Gardens

Botanical garden is an institution located in an enclosed piece of land which grows numerous kinds of plants obtained from different places for botanical studies. Each plant is first identified and then labelled indicating its botanical/scientific name and its family.

#### List of some Botanical Gardens

- (i) Royal Botanical Garden, Kew (England)
- (ii) National Botanical Garden, Lucknow
- (iii) Indian Botanical Garden, Howrah
- (iv) Lloyd Botanical Garden, Darjeeling

### 15. Museum

- Museum is a place used for storing, preservation and exhibition of both plants and animals. All educational institutes and universities maintain museums in their Botany and Zoology departments.
- A commonly used preservative solutions is “Formalin”. Plants and animals specimen may also be preserved as dry specimens. For instances, insects are collected, killed and pinned before preserving them in special insect boxes while larger animals like reptiles, birds and mammals are usually stuffed and then preserved.
- National Museum of Natural History (NMNH) in Delhi is important from natural science point of view.

## 16. Zoological Parks

- Zoological parks commonly known as zoos are the places where live wild animals are kept in protected environment which is made similar to their natural habitats as much as possible. Here, they are provided with protection and care by human beings.
- These parks serve as ideal means to study and learn different food habits and behaviour of variety of animals. So, students should visit nearby zoos for knowledge and entertainment both.
- National Zoological Park (Delhi) is one of the finest zoos of Asia.

## 17. Key

- Key is an important taxonomic aid used for identification of plants and animals based on the **similarities** and **dissimilarities**. Actually, it is a set of alternate characters of different types arranged sequence wise in such a fashion that by selection and elimination one can quickly find out the name of the organism.
- The keys are based on the set of contrasting characters generally in a pair known as “**couplet**”. Each character of the couplet or statement in the key is called as “**lead**”.
- One has to choose correct option between two statements of characters of definite species so that the animal or plant is identified accurately. **Keys are generally analytical in nature.**

## 18. Flora, Manuals, Monographs and Catalogues

- These flora, manuals, monographs, etc. are recorded descriptions of plants, animals and other organisms. They provide correct identification and description of variety of living organisms.
  - ❖ **Flora** : It is a book containing information about plants found in a particular area. It gives the actual account of habitat and distribution of various plants of a given area. These provide the index to the plant species found in a particular area. For example, Flora of Delhi by J.K. Maheshwari.
  - ❖ **Manual** : It is a book containing complete listing and description of the plants growing in a particular area. They provide useful information for identification of names of various species found in an area.
  - ❖ **Monograph** : It contains information of any one taxon.
  - ❖ **Catalogue** : It include the alphabetical arrangement of species of a particular place describing their features.

## CBSE Exam Pattern Exercise

### Subjective Questions (1)

#### (Q1 to 3) One Mark

1. Write the correct order of sequence of taxonomical categories.
2. What do you mean by chemotaxonomy?
3. Define genus

#### (Q4 to 6) Two Marks

4. Elaborate about keys as one of taxonomical aids?
5. What is classical taxonomy and modern taxonomy?
6. What is meant by living ? Give any four defining features of all life forms.

#### (Q7 to 8) Three Marks

7. What are taxonomical aids? Give importance of herbaria & museums . How are botanical gardens and zoological parks useful in conserving biodiversity?
8. Metabolism is a defining feature of all living organisms without exception. Isolated metabolic reactions in vitro are not living things but surely living reactions. Comment

#### (Q9 to 10) Five Marks

9. ICBN has provided a code for classification of plants. Give hierarchy of units of classification botanists follow while classifying plants.
10. What is binomial system of nomenclature? Who proposed this system? Why is binomial nomenclature the most acceptable mode of naming organism?



# Answer & Solution

Q1

Species → genus → family → order → class → Phylum → kingdom

Q2

Chemotaxonomy is based on the presence or absence of certain chemicals in cells or tissues.

Q3

Terms 'Genus' was coined by Tournefort. Genera are the aggregates of closely related species. For example, Potato (*Solanum tuberosum*), tomato (*Solanum lycopersicum*) and brinjal (*Solanum nigrum*) are three different

Q4

It is a taxonomical aid used for the identification of plants and animals. The keys are based on the paired contrasting characters called as couplet. It represents two alternate forms of a given character. Keys are generally of two types

(i) **Indented or yoked keys** It has a sequence of forms of different characters.

(ii) **Bracketed keys** It includes one pair of contrasting statements for a given character.

Separate taxonomic keys are required for each taxonomic category such as family, genus and species for identification purposes.

Q5

1. Classical Taxonomy deals with morphospecies. Modern Taxonomy deals with biological species.

2. Classical Taxonomy has typological concept. Modern Taxonomy has a population or biosystematic concept.

3. In Classical taxonomy, species is considered to be static. In modern taxonomy, species is considered to be dynamic.

4. Classical taxonomy does not study evolution and inter-relationships of species. Modern Taxonomy studies primitiveness, advancement and inter-relationships of species.

Q6

The living organism exhibit distinctive characteristics, which are as follows (i) Growth All living organisms grow in size as well as in number. Plants show growth all through their life whereas animals up to a certain growth period after which growth ceases. (ii) Reproduction The process of

reproduction is essential for the continuity of life on earth. Every organism whether unicellular or multicellular gives rise to an individual of its own kind. Lower organisms usually reproduce asexually, e.g., Hydra, fungi, yeast, etc. Sexual reproduction is found in advanced group of organisms, which involves two parents. In Amoeba growth and reproduction however is synonymous. (iii) Metabolism There are thousands of enzymatic ally controlled reactions occurring in all living cells. These are synthesizing (anabolic) and breaking down (catabolic) reactions. Hence, it is the most important characteristic of living organisms. (iv) Response to stimuli Consciousness and response to stimulus is the defining property of all living organisms. Plants respond to light, water temperature. Unicellular organisms also sense their environment and respond accordingly. (v) Cellular organization The cellular organization of the body is the defining feature of life forms. Cells work together in hierarchical manner group of cells make tissues, tissues make organs, organs make systems, systems when work in co-ordination form an individual. So, we can say living organisms are self replicating, evolving and capable of responding to external stimuli.

Q7

The taxonomic aids are the aids which help in identification, classification and naming of a newly discovered organisms (plant or animal). It could be in the form of preserved document like herbaria or specimen kept at museums or scientific institutions. Other aids can be in the form of written document like monograph ,taxonomic keys, couplets, etc. A new organism found can be studied while comparing it with living plants and animals living in protected areas like Botanical gardens, Zoological parks, etc. Botanical gardens helps in conservation of plants by (i) Growing important local plant species and keeping record of them. (ii) Growing and maintaining rare and endangered species. (iii) Supplying seeds for different, aspects of botanical research. Whereas zoological parks also contribute in conserving biodiversity by (i) Providing natural environment and open space to animals, i.e., wild life species. (ii) Keeping them safe from their predators ensuring protection, food and shelter. (iii) Providing home to different native and exotic wild animals. (iv) Involving in the rescue of endangered species. (v) Facilitating breeding of animals and releasing them free. Thus, both botanical gardens and zoological parks play an important role in conservation of biodiversity.

Q8

All living organisms are made of chemicals which belong to various classes and have different sizes and functions. In the cells, these chemicals are constantly being made and changed into some other biomolecules. These conversions or chemical reactions taking place in the body are collectively known as metabolism. All plants, animals, fungi and microbes exhibit metabolism. Non-living objects do not exhibit metabolism. So we can say metabolism is a defining feature of all living organisms without exception.

Metabolic reactions can be demonstrated outside the body in a test-tube. A metabolic reaction performed in a test-tube (in vitro) is neither living nor non-living but surely living reaction.

Q9

The study of taxonomy has led to the taxonomic categories – Kingdom, phylum, class, order, family, genus, and species. Now let us see how all the organisms are classified into the hierarchy.

### Species

It is the lowest level of classification and shows the high level of similarities among the organisms. One species can be distinguished from other closely related species based on distinct differences in morphology. Let us look at an example; Plasmodium vivax and Plasmodium falciparum – Both are malaria-causing parasites but have different effects on a patient. Plasmodium is the name of the genus and

## 01 Units and Measurement

has a number of species which show distinct morphological characteristics.

### Genus

This taxonomic group comprises several species which have similar characteristics but different from that of species from another genus. Considering the same example of Plasmodium, it is a genus with multiple species which are similar to each other and differs from the species of another genus.

### Family

This category of taxonomy includes various genus which shares some resemblance among themselves. However, the number of similarities decrease compared to species and genus. For example, the genus of tiger, leopard, lion, i.e., Panthera and the genus of cats i.e., Felis are grouped together in the family Felidae.

### Order

The classification which begins with the order has fewer comparisons as an effect, they are grouped based on aggregates of characteristics. A group of families showing somewhat few similarities forms an order. For example, the order of carnivores i.e., Carnivore includes families like Felidae and Canidae.

### Class

A group of Orders which share a few similarities forms a Class. For example- Orders of primates and carnivores are grouped together in the Class of mammals.

### Phylum

This is the next level of classification as along a number of Classes are clubbed up to form one Phylum. Example – aves, fishes, reptiles, mammals, and amphibians combined to form the phylum of or Chordata.

### Kingdom

This is the highest level of classification. We have Kingdoms like Plantae, Animalia, Fungi, Protista, and Monera.

Q10

**Binomial nomenclature**, introduced by Carolus Linnaeus is the method of naming an organism with the genus name first and species name later.

### Conventions followed while naming

- \*Name of the genus begins with a capital letter.
- \*Name of the species should begin with a small letter.
- \*Scientific name should be in Italics when printed.
- \*Genus name and the species name should be underlined separately while handwritten.

The advantages of scientific over common names are that they are accepted by speakers of all languages, that each name applies only to one species, and that each species has only one name.



CLASS 11

# CBSE

# CHEMISTRY

- ▶ Specially designed eBook for complete CBSE syllabus
- ▶ CBSE preparation strategy & direction, speedy revision
- ▶ Chapter-wise important Problem-Solving
- ▶ Theory, Formulas & Diagrams to crack CBSE
- ▶ Updated New Pattern Objective questions
- ▶ Detailed high quality video lectures by experienced faculties
- ▶ CBSE full preparation for Class XI & XII



**misostudy**  
connect smart learning

**JEE·NEET·AIIMS·CBSE·FOUNDATION**  
[www.misostudy.com](http://www.misostudy.com)

☎ 8929803804 (MON-FRI: 9am-6pm) [support@misostudy.com](mailto:support@misostudy.com)

MISO STUDY INDIA PVT. LTD.  
2ND FLOOR 65-A, OMPRO TOWER, KALU SARAI, NEW DELHI, DELHI 110016

# Class 11 | Chemistry

## 02 Structure of Atom



Misostudy.com



### Scientific eBook

This is a sample eBook. The eBook provides chapter-wise theories and examples, formulas, diagrams, and Exam pattern Problem-Solving of the full syllabus.



### Complete video lectures

The theory and Problem-Solving in this eBook provide specially designed the video lectures by experienced faculties. Get Free trial and preview on Misostudy.com.

## 01. Introduction

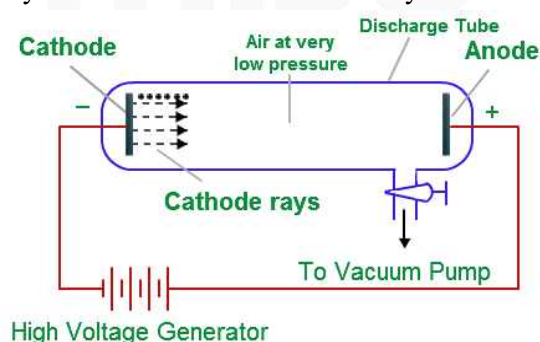
**John Dalton 1808**, believed that matter is made up of extremely minute indivisible particles, called atom which takes part in chemical reactions. These particle can neither be created nor be destroyed. However, modern researches have conclusively proves that atom is no longer an indivisible particle. Modern structure of atom is based on Rutherford's scattering experiment, quantization of energy and wave mechanical model.

### Composition of Atom

The works of J.J. Thomson and Ernst Rutherford actually laid the foundation of the modern picture of the atom. It is now verified that the atom consists of several sub-atomic particles like electron, proton, neutron, positron, neutrino, meson etc. Out of these particles the electron, proton and the neutron are called fundamental subatomic particles.

## 02. ELECTRON ( ${}_{-1}e^0$ , e)

Electron was discovered by J.J. Thomson(1897) and it is a negatively charged particle. Cathode rays were discovered by William Crooke & J.J. Thomson using a cylindrical hard glass tube fitted with two metallic electrodes. This tube was known as discharge tube. They passed electricity (10,000V) through a discharge tube at very low pressure. Blue rays emerged from the cathode. These rays were termed as Cathode rays.



### Properties of Cathode rays

- (i) Cathode rays travel in straight line.
- (ii) Cathode rays produce mechanical effect, as they can rotate the wheel placed in their path.
- (iii) Cathode rays consist of negatively charged particles known as electron.

## 02 Structure of Atom

- (iv) Cathode rays travel with high speed.
- (v) Cathode rays can cause fluorescence.
- (vi) Cathode rays heat the object on which they fall due to transfer of kinetic energy to the object.
- (vii) When cathode rays fall on heavy metals, X-rays are produced.
- (viii) Cathode rays possess ionizing power i.e., they ionize the gas through which they pass.
- (ix) The cathode rays produce scintillation on the photographic plates.
- (x) They can penetrate through thin metallic sheets.
- (xi) The nature of these rays does not

### 03. Thomson's Model of Atom [1904]

- Thomson was the first to propose a detailed model of the atom.
- Thomson proposed that an atom consists of a uniform sphere of positive charge in which the electrons are distributed more or less uniformly.
- This model of atom is known as “Plum-Pudding model” or “Raisin Pudding Model” or “Water Melon Model”.

#### Drawbacks:

- An important drawback of this model is that the mass of the atoms is considered to be evenly spread over that atom.
- It is a static model. It does not reflect the movement of electron.
- It could not explain the stability of an atom.

### 04. Rutherford's Scattering Experiment

#### Rutherford observed that

- (i) Most of the  $\alpha$ -particles (nearly 99.9%) went straight without suffering any deflection.
- (ii) A few of them got deflected through small angles.
- (iii) A very few (about one in 20,000) did not pass through the foil at all but suffered large deflections (more than  $90^\circ$ ) or even came back in the direction from which they have come i.e. a deflection of  $180^\circ$ .

#### Following conclusions were drawn from the above observations-

- (i) Since most of the  $\alpha$ -particle went straight through the metal foil undeflected, it means that there must be very large empty space within the atom.
- (ii) Since few of the  $\alpha$ -particles were deflected from their original paths through moderate angles; it was concluded that whole of the +ve charge is concentrated and the space occupied by this positive charge is very small in the atom.
  - When  $\alpha$ -particles come closer to this point, they suffer a force of repulsion and deviate from their paths.
  - The positively charged heavy mass which occupies only a small volume in an atom is called nucleus. It is supposed to be present at the centre of the atom.

- (iii) A very few of the  $\alpha$ -particles suffered strong deflections or even returned on their path indicating that the nucleus is rigid and  $\alpha$ -particles recoil due to direct collision with the heavy positively charged mass.

#### Drawbacks of rutherford model-

- (i) This theory could not explain stability of atom. According to Maxwell electron loose its energy continuously in the form of electromagnetic radiations. As a result of this, the e- should loose energy at every turn and move close and closer to the nucleus following a spiral path. The unlimited result will be that it will fall into the nucleus, thereby making the atom unstable.
- (ii) If the electrons loose energy continuously, the observed spectrum should be continuous but the actual observed spectrum consists of well defined lines of definite frequencies. Hence the loss of energy by electron is not continuous in an atom.

#### 05. Distance of closest approach :

When the  $\alpha$ -particles approaches the nucleus to made a head-on collision with a the nucleus, the  $\alpha$ -particle approaches the nucleus until coulombic potential energy of repulsion,  $k \frac{Z_1 Z_2 e^2}{r}$ ,

becomes equal to its initial K.E.,  $\frac{1}{2} m.v^2$ .

$$\text{Thus } \frac{1}{2} mv^2 = k \frac{Z_1 Z_2 e^2}{r}$$

$$\text{Hence, the distance of closest approach, } r = \frac{k z_1 z_2 e^2}{\left(\frac{1}{2}mv^2\right)}$$

The nucleus must be further smaller than the distance of closest approach.

#### 06. Moseley Experiment (Discovery of Atomic Number)

Moseley (1912-1913), investigated the X-rays spectra of 38 different elements, starting from aluminium and ending in gold. He measured the frequency of principal lines of a particular series (the  $\alpha$ -lines in the K series) of the spectra. It was observed that the frequency of a particular spectral line gradually increased with the increase of atomic mass of the element. But, it was soon realised that the frequency of the particular spectral line was more precisely related with the serial number of the element in the periodic table which he termed as atomic number (Z). He presented the following relationship:

$$\sqrt{\nu} = a(Z-b)$$

where,  $\nu$ =frequency of X-rays,  $Z$ =atomic number, 'a' and 'b' are constants. When the values of square root of the frequency were plotted against atomic number of the elements producing X-rays, a straight line was obtained.

## 07. Some Atomic Terms

**Atomic number** = Number of unit positive charge on nucleus = Number of protons in nucleus  
= Number of electrons in neutral atom.

Two different elements can never have identical atomic number.

**Mass number(A)** = Number of protons + Number of neutrons or Atomic number (Z)

Hence Number of neutrons =  $A - Z$ .

The atom of an element X having mass number (A) and atomic number (Z) may be represented by a symbol,

**Isotopes** : Atoms of an element with the same atomic number but different mass number.

eg.  ${}_1\text{H}^1$                        ${}_1\text{H}^2$                        ${}_1\text{H}^3$   
Protonium                      deuterium                      Tritium

**Isodiapheres** : The elements which have same value of  $(n - p)$  is called Isodiapheres.

eg.  ${}_7\text{N}^{14}$                        ${}_8\text{O}^{16}$   
Values of  $(n - p)$                       0                      0

**Isotone** : Elements which contain same no. of neutron is called **Isotone**.

eg.  ${}_{14}\text{Si}^{30}$                        ${}_{15}\text{P}^{31}$                        ${}_{16}\text{S}^{32}$   
number of neutrons                      16                      16                      16

**Isobar** : The two different atoms which have same mass number but different atomic number is called Isobar.

eg.  ${}_{18}\text{Ar}^{40}$                        ${}_{19}\text{K}^{40}$                        ${}_{20}\text{Ca}^{40}$

**Isoelectronic** : Ion or atom or molecule which have the same mass number of electron is called Isoelectronic species.

eg.  ${}_{17}\text{Cl}^-$                        ${}_{18}\text{Ar}$                        ${}_{19}\text{K}^+$                        ${}_{19}\text{Ca}^{+2}$   
No. of electrons                      18                      18                      18                      18

## 08. Bohr's Atomic Model

This model was based on the quantum theory of radiation and the classical law of physics. It gave new idea of atomic structure in order to explain the stability of the atom and emission of sharp spectral lines.

### Postulates

- The atom has a central massive core nucleus where all the protons and neutrons are present. The size of the nucleus is very small.
- The electron in an atom revolve around the nucleus in certain discrete orbits. Such orbits are known as stable orbits or non – radiating or stationary orbits.
- An electron can move only in those permissive orbits in which the angular momentum ( $mvr$ ) of the electron is an integral multiple of  $h/2\pi$  Thus,

$$mvr = n \frac{h}{2\pi}$$

Where,  $m$  = mass of the electron,  $r$  = radius of the electronic orbit,  $v$  = velocity of the electron in its orbit.

- (iv) The angular momentum can be  $\frac{h}{2\pi}$ ,  $\frac{2h}{2\pi}$ ,  $\frac{3h}{2\pi}$ , .....  $\frac{nh}{2\pi}$ . This principal is known as quantization of angular momentum. In the above equation 'n' is positive integer which has been called as principal quantum number. It can have the values  $n=1,2,3,\dots$  (from the nucleus). Various energy levels are designed as  $K(n=1)$ ,  $L(n=2)$ ,  $M(n=3),\dots$ etc. Since the electron present in these orbits is associated with some energy, these orbits are called energy levels.
- (v) The emission or absorption of radiation by the atom takes place when an electron jumps from one stationary orbit to another.

### 09. Calculation of the radius of the Bohr's orbit :

Suppose that an electron having mass 'm' and charge 'e' revolving around the nucleus of charge 'Ze' (Z is atomic number & e is charge) with a tangential/linear velocity of 'v'. Further consider that 'r' is the radius of the orbit in which electron is revolving. According to Coulomb's law, the electrostatic force of attraction (F) between the moving electron and nucleus is -

$$F = \frac{KZe^2}{r^2}$$

Where :  $K = \text{constant} = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

and the centripetal force  $F = \frac{mv^2}{r}$

Hence  $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$  or,  $v^2 = \frac{KZe^2}{mr} \dots(1)$

From the postulate of Bohr,  $mvr = \frac{nh}{2\pi}$  or,  $v^2 = \frac{n^2h^2}{4\pi^2m^2r^2} \dots(2)$

From equation (1) and (2) :

$$\therefore r = \frac{n^2h^2}{4\pi^2mKZe^2}$$

On putting the value of e, h, m,

$$r = 0.529 \times \frac{n^2}{Z} \text{ \AA}$$

### 10. Calculation of velocity of an electron in Bohr's orbit

Velocity of the revolving electron in  $n^{\text{th}}$  orbit is given by

$$mvr = \frac{nh}{2\pi} \quad v = \frac{nh}{2\pi mr}$$

Putting the value of r in above equation

$$\text{then } v = \frac{nh \times 4\pi^2 mZe^2}{2\pi mn^2 h^2}, \quad v = \frac{2\pi Ze^2}{nh}$$

on putting the values of  $e$  and  $h$ ,

$$v = 2.188 \times 10^6 \times \frac{Z}{n} \text{ m/sec}$$

### 11. Calculation of energy of an electron :

The total energy of an electron revolving in a particular orbit is -

$$\text{T. E.} = \text{K. E.} + \text{P. E.}$$

$$\text{The K.E. of an electron} = \frac{1}{2}mv^2$$

$$\text{and the P.E. of an electron} = -\frac{KZe^2}{r}$$

$$\text{Hence, T.E.} = \frac{1}{2}mv^2 - \frac{KZe^2}{r} \quad \dots(3)$$

$$\text{But } \frac{mv^2}{r} = \frac{KZe^2}{r^2} \quad \text{or } mv^2 = \frac{KZe^2}{r}$$

Substituting the value of  $mv^2$  in the equation (3)

$$\text{T.E.} = \frac{KZe^2}{2r} - \frac{KZe^2}{r} = -\frac{KZe^2}{2r}$$

$$\text{So, T.E.} = -\frac{KZe^2}{2r}$$

Substituting the value of 'r' in the equation of T.E. .

$$E = -\frac{kZe^2}{2} \times \frac{4\pi^2 Ze^2 mk}{n^2 h^2} = -\frac{2\pi^2 Z^2 e^4 mk^2}{n^2 h^2}$$

Thus, the total energy of an electron in  $n^{\text{th}}$  orbit is given by

$$E_n = \frac{2\pi^2 Z^2 e^4 mk^2}{n^2 h^2}$$

$$= -13.6 \times \frac{Z^2}{n^2} \text{ eV/atom}$$

$$= -21.8 \times 10^{-19} \times \frac{Z^2}{n^2} \text{ J/atom}$$

$$= -313.6 \times \frac{Z^2}{n^2} \text{ Kcal/mole}$$

### 12. Relation between P. E., K. E. & T. E. :

$$\text{P.E.} = -\frac{kZe^2}{r}, \quad \text{K.E.} = \frac{1}{2} \frac{kZe^2}{r}, \quad \text{T.E.} = -\frac{1}{2} \frac{kZe^2}{r},$$

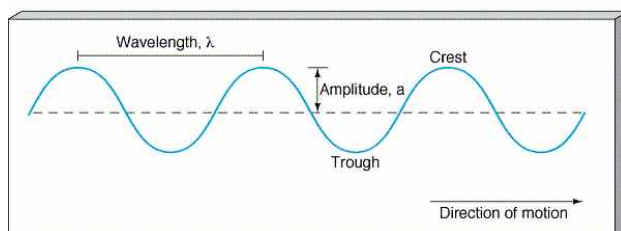


$$T.E. = \frac{P.E.}{2} = -K.E.$$

### 13. Details About Waves

A wave motion is a means of transferring energy from one point to another point without any actual transportation of matter between these points. When we throw a piece of stone on the surface of water in a pond, we observe circles of ever increasing radius, till they strike the wall of the pond.

### 14. Some important terms related with wave motion.

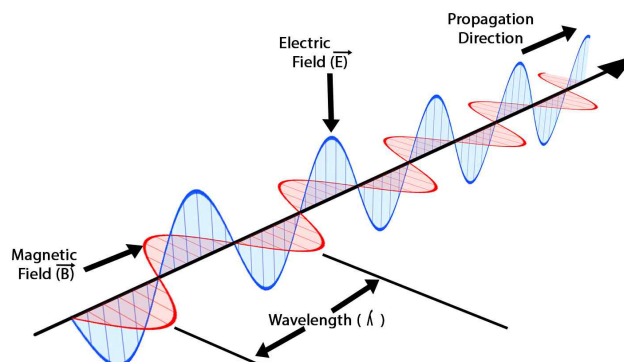


- (i) **Wave length ( $\lambda$ ):** The distance between two adjacent crest or trough of the wave (or the distance between two similar neighbouring points)
- (ii) **Time period (T):** Time for one complete oscillation of wave is called the period (T). Time taken by the wave to travel a distance equal to one wavelength. If  $C$  is the speed of wave, then  $C = \frac{\lambda}{T}$ .
- (iii) **Frequency ( $\nu$ ):** Number of oscillations per unit time is called frequency.  $\nu = \frac{C}{\lambda}$
- (iv) **Wave number ( $\bar{\nu}$ ):** Number of wavelength per unit length.  $\bar{\nu} = \frac{1}{\lambda}$
- (v) **Amplitude (A):** It is the height of crest or depth of a trough of a wave.

### 15. Characteristics of electromagnetic radiations:

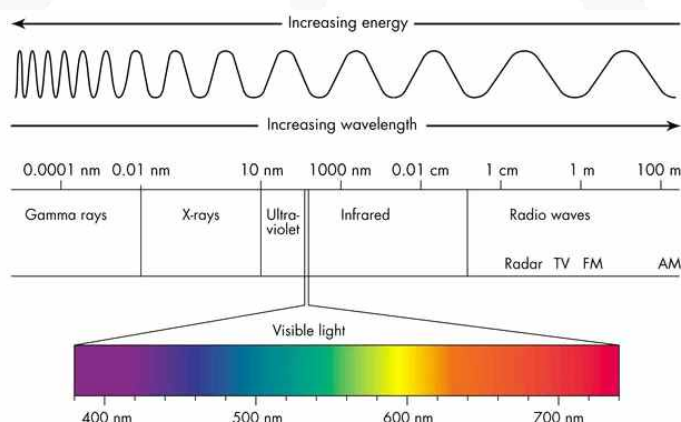
- (i) All electromagnetic waves move or travel with the same speed equal to that of light.
- (ii) They do not require any medium to propagate.
- (iii) These consist of electric and magnetic field that oscillate in the direction perpendicular to each other and to the direction in which the wave propagates (as shown in above diagram)

## Electromagnetic Wave



## 16. Electromagnetic Spectrum

Arrangement of various types of electromagnetic radiations in order of their increasing (or decreasing) wavelengths or frequencies is known as electromagnetic spectrum.



## 17. Planck's Quantum Theory

According to this theory atoms or molecules could emit or absorb energy only in discrete quantities (small packets) and not in any arbitrary amount. Planck gave the name quantum to the smallest quantity of energy that can be emitted in the form of E.M. radiation.

The energy of a photon is proportional to its frequency and is given by

$$E = h\nu \text{ where } h = 6.626 \times 10^{-34} \text{ J sec}$$

A body can emit or absorb energy only in terms of the integral multiples of quantum, i.e.

$$E = n \cdot h\nu, \text{ where } n = 1, 2, 3, \dots$$

i.e. a body can emit or absorb energy as  $h\nu$ ,  $2h\nu$  .... but it can not emit or absorb energy in fractional values of  $h\nu$  such as  $1.5 h\nu$ ,  $2.5 h\nu$ .

Einstein supported the planck's theory and explained the photoelectric effect considering that electromagnetic radiations also propagate in the form of photon. Energy of each photon

depends on frequency of light  $\left(E = h\nu = \frac{hc}{\lambda}\right)$ .

Since wave character of light explains the interference and diffraction phenomenon while the particle character explains, black body radiations and photoelectric effect, the light was considered to have wave as well as particle character [Dual character of light.]

- (i) Wave nature : diffraction, interference, polarisation.
- (ii) Particle nature : photoelectric effect.

## 18. Hydrogen Spectrum

Hydrogen spectrum is an example of line or atomic emission spectrum. When an electric discharge is passed through hydrogen gas at low pressure, a bluish light is emitted. All these lines of H-spectrum have Lyman, Balmer, Paschen, Barckett, Pfund and Humphrey series. These spectral series were named by the name of scientist discovered them. To evaluate wavelength of various H-lines Rydberg introduced the following expression,

$$\frac{1}{\lambda} = \frac{1}{\lambda} = \frac{\nu}{c} = R \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Where R is constant known as Rydberg's constant its value is  $109,67800 \text{ m}^{-1}$

Although H-atom consists only one electron yet it's spectra consist of many spectral lines because electrons in different hydrogen atoms absorb different amount of energies and are excited to different energy levels. Since life time of electrons in these excited states is very small, they return to some lower energy level or even to the ground state in one or more jumps.

Maximum number of lines produced when an electron jumps from  $n$ th level to ground level =  $\frac{n(n-1)}{2}$ .

## 19. Lyman Series

- (a) It is a first series of spectral series of H.
- (b) It's value of  $n_1 = 1$  and  $n_2 = 2,3,4$ , where ' $n_1$ ' is ground state and ' $n_2$ ' is called excited state of electron present in H-atom.
- (c) If the electron goes to  $n_1 = 1$  from  $n_2 = 2$  — first Lyman series  
If the electron goes to  $n_1 = 1$  from  $n_2 = 3$  — Second Lyman series  
If the electron goes to  $n_1 = 1$  from  $n_2 = 4$  — third Lyman series ----- so on.

$$(d) \frac{1}{\lambda} = R_H \left[ \frac{1}{1^2} - \frac{1}{n_2^2} \right] \text{ where } n_2 > 1 \text{ always.}$$

$$(e) \text{ For marginal line of limiting line } n_2 = \infty. \text{ Hence the wavelength of marginal line} = \frac{n_1^2}{R_H}$$

for all series. So, for Lyman series it is  $\frac{1}{R_H}$ .

## 20. Balmer Series :

- (i) It is second series of H-spectral series.
- (ii) It was found out in 1892 in visible region by **Balmer**.
- (iii) Balmer series was found out before all series because it was found in visible region.
- (iv) It's value of  $n_1 = 2$  and  $n_2 = 3, 4, 5, \dots$
- (v) If the electron goes to  $n_1 = 2$  from  $n_2 = 3$  — first Balmer series  
If the electron goes to  $n_1 = 2$  from  $n_2 = 4$  — Second Balmer series  
If the electron goes to  $n_1 = 2$  from  $n_2 = 5$  — third Balmer series ----- so on.

$$(vi) \text{ The wavelength of marginal line of Balmer series} = \frac{n_1^2}{R_H} = \frac{2^2}{R_H} = \frac{4}{R_H}.$$

$$(vii) \frac{1}{\lambda} = R_H \left[ \frac{1}{2^2} - \frac{1}{n_2^2} \right] \text{ where } n_2 > 2 \text{ always.}$$

## 21. Paschen Series :

- (i) It is the third series of H – spectrum.
- (ii) It was found out in infra red region by **Paschen**.
- (iii) It's value of  $n_1 = 3$  and  $n_2 = 4, 5, 6, \dots$
- (iv) If the electron goes to  $n_1 = 3$  from  $n_2 = 4$  — first Paschen series  
If the electron goes to  $n_1 = 3$  from  $n_2 = 5$  — Second Paschen series  
If the electron goes to  $n_1 = 3$  from  $n_2 = 6$  — third Paschen series ----- so on.

$$(v) \text{ The wavelength of marginal line of paschen series} = \frac{n_1^2}{R_H} = \frac{3^2}{R_H} = \frac{9}{R_H}.$$

$$(vi) \frac{1}{\lambda} = R_H \left[ \frac{1}{3^2} - \frac{1}{n_2^2} \right] \text{ where } n_2 > 3 \text{ always.}$$

## 22. Brackett Series :

- (i) It is fourth series of H – spectrum.
- (ii) It was found out in infra red region by **Brackett**.
- (iii) It's value of  $n_1 = 4$  and  $n_2 = 5, 6, 7, \dots$

- (iv) If the electron goes to  $n_1 = 4$  from  $n_2 = 5$  — first Brackett series  
 If the electron goes to  $n_1 = 4$  from  $n_2 = 6$  — Second Brackett series  
 If the electron goes to  $n_1 = 4$  from  $n_2 = 7$  — third Brackett series ----- so on.
- (v) The wavelength of marginal line of Brackett series =  $\frac{n_1^2}{R_H} = \frac{4^2}{R_H} = \frac{16}{R_H}$ .
- (vi)  $\frac{1}{\lambda} = R_H \left[ \frac{1}{4^2} - \frac{1}{n_2^2} \right]$  where  $n_2 > 4$  always.

### 23. Pfund Series :

- (i) It is fifth series of H – spectrum.  
 (ii) It was found out in infra red region by Pfund.  
 (iii) It's value of  $n_1 = 5$  and  $n_2 = 6, 7, 8$  .....
- (iv) If the electron goes to  $n_1 = 5$  from  $n_2 = 6$  — first Pfund series  
 If the electron goes to  $n_1 = 5$  from  $n_2 = 7$  — Second Pfund series  
 If the electron goes to  $n_1 = 5$  from  $n_2 = 8$  — third Pfund series ----- so on.
- (v) The wavelength of marginal line of Pfund series =  $\frac{n_1^2}{R_H} = \frac{5^2}{R_H} = \frac{25}{R_H}$ .
- (vi)  $\frac{1}{\lambda} = R_H \left[ \frac{1}{5^2} - \frac{1}{n_2^2} \right]$  where  $n_2 > 5$  always.

### 24. Calculation of Rydberg Constant from bohr's atomic model

Suppose that an electron transit from first energy. level to second energy level. Then, the change of energy. is given by

$$\Delta E = E_{n_2} - E_{n_1}$$

$$\text{or, } = \left[ \frac{-2\pi^2 m Z^2 e^4 k^2}{n_2^2 h^2} \right] - \left[ \frac{-2\pi^2 m Z^2 e^4 k^2}{n_1^2 h^2} \right] \quad \text{or, } \frac{hc}{\lambda} = \frac{-2\pi^2 m Z^2 e^4 k^2}{h^2} \times \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{or, } \frac{1}{\lambda} = \frac{2\pi^2 m Z^2 e^4 k^2}{ch^3} \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = R_H \cdot Z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{or, } R_H = \frac{2\pi^2 m e^4 k^2}{ch^3} \quad \text{Rydberg constant}$$

### 25. Particle and Wave Nature of Electron

In 1924, de Broglie proposed that an electron, like light, behaves both as material particle and as a wave. This proposal gave a new theory, known as wave mechanical theory of matter. According to this theory, the electrons, protons and even atoms, when in motion, possess wave properties.

de Broglie derived an expression for calculating the wavelength of the wave associated with the electron. According to Planck's equation

$$E = h\nu = h \cdot \frac{c}{\lambda} \quad \dots(1)$$

The energy of a photon on the basis of Einstein's mass-energy relationship is

$$E = mc^2 \quad \dots(2)$$

Where,  $c$  is the velocity of the electron.

From (1) and (2) equation

$$h \times \frac{c}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{p} \quad \dots(3)$$

Momentum of the moving electron is inversely proportional to its wavelength.

Let kinetic energy of the particle of mass 'm' is  $E$ .

$$E = \frac{1}{2}mv^2$$

$$2Em = m^2v^2$$

$$\sqrt{2Em} = mv = p \text{ (momentum)}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Em}} \quad \dots(4)$$

Let a charged particle, with charge  $q$  be accelerated with a potential of  $V$ ; then the kinetic energy may be given as :  $E=qV$

$$\frac{1}{2}mv^2 = qV$$

$$m^2v^2 = 2qVm$$

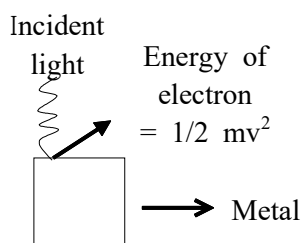
$$mv = \sqrt{2qVm}$$

$$\lambda = \frac{h}{\sqrt{2qVm}} \quad \dots(5)$$

and,

## 26. Photoelectric Effect

Emission of electrons from a metal surface when exposed to light radiations of appropriate wavelength is called photoelectric effect. The emitted electrons are called photoelectrons.



Work function or threshold energy may be defined as the minimum amount of energy required to eject electrons from a metal surface. According to Einstein,

Maximum kinetic energy of the ejected electron = absorbed energy – work function

$$\frac{1}{2} mv_{\max}^2 = h\nu - h\nu_0 = hc \left[ \frac{1}{\lambda} - \frac{1}{\lambda_0} \right]$$

where,  $\nu_0$  and  $\lambda_0$  are threshold frequency and threshold wavelength respectively.

### Stopping Potential

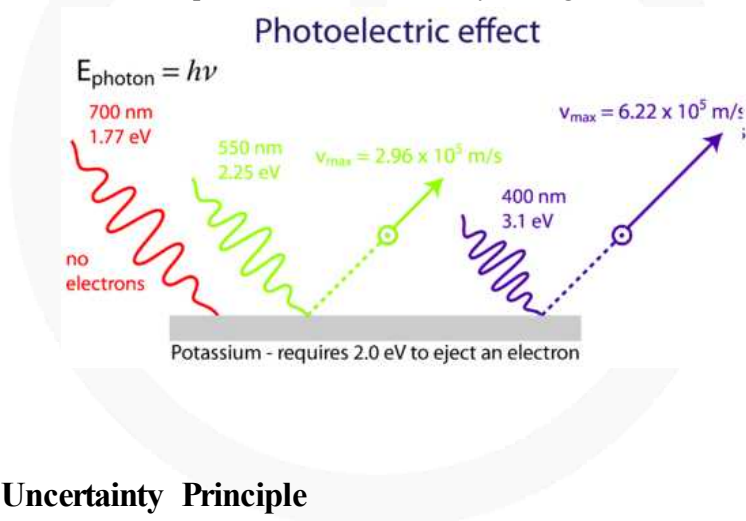
The minimum potential at which the photoelectric current becomes zero is called stopping potential.

If  $V_0$  is the stopping potential, then

$$eV_0 = h(\nu - \nu_0) \text{ or } V_0 = \frac{h(\nu - \nu_0)}{e} = \frac{\text{K.E.}_{\max}}{e}$$

### Some facts of Photoelectric Effect

- There is no time lag between incidence of light and emission of photoelectrons.
- For emission of photoelectrons, the frequency of incident light must be equal to or greater than the threshold frequency.
- Rate of emission of photoelectrons from a metal surface is directly proportional to the intensity of incident light.
- The maximum kinetic energy of photoelectrons depends on the frequency of incident radiation; but, it is independent of the intensity of light used.



## 27. Heisenberg's Uncertainty Principle

According to this principle it is impossible to measure simultaneously the exact position and exact momentum of a body as small as an electron. If uncertainty of measurement of position is  $\Delta x$  and uncertainty of measurement of momentum is  $\Delta p$  or  $m\Delta v$ , then according to Heisenberg,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \quad \text{or} \quad \Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

where  $h$  is Planck's constant

Like de Broglie equation, this principle has significance only for microscopic particles.

## 28. Wave Mechanical Model of Atom

The atomic model which is based on the particle and wave nature of the electron is known as wave mechanical model of the atom. This was developed by Erwin Schrodinger in 1926. This model describes the electron as a three-dimensional wave in the electronic field of positively charged nucleus.

### Significance of $\Psi$ :

The wave function may be regarded as the amplitude function expressed in terms of coordinates  $x$ ,  $y$  and  $z$ . The wave function may have positive or negative values depending upon the values of coordinates.

### Significance of $\Psi^2$ :

$\Psi^2$  is a probability factor. It describes the probability of finding an electron within a small space. The space in which there is maximum probability of finding an electron is termed as orbital.

## 29. Quantum Numbers

Quantum numbers are used to specify and display complete information about size, shape and orientation of the orbital. These are principal, azimuthal and magnetic quantum number, which follow directly from solution of Schrodinger wave equation.

## 30. Principal Quantum Number ( $n$ ) :

- (i) It was proposed by Bohr and denoted by 'n'.
- (ii) It determines the average distance between electron and nucleus, means it is denoted the size of atom.
- (iii) It determines the energy of the electron in an orbit where electron is present.
- (iv) The maximum number of an electron in an orbit represented by this quantum number as  $2n^2$ .

## 31. Azimuthal Quantum Number or Angular Quantum Number ( $l$ ) -

- (i) It was proposed by Sommerfeld and denoted by 'l'.
- (ii) It determines the number of subshells or sublevels to which the electron belongs.
- (iii) It tells about the shape of subshells.
- (iv) It also expresses the energies of subshells  $s < p < d < f$  (Increasing energy).
- (v) The value of  $l$  is integral values upto  $(n-1)$ , starting from zero where 'n' is the number of principle shell.



(vi)

Value of $l$	0	1	2	3
Name of subshell	s	p	d	f
Shape of orbital	spherical	Dumbbell	Clover leaf (Except $d_{z^2}$ doughnut)	Complex

(vii) It represent the orbital angular momentum, which is equal to  $\frac{h}{2\pi} \sqrt{l(l+1)}$ .(viii) The number of electrons in subshell =  $2(2l+1)$ .

(ix) For a given value of 'n' the total value of 'l' is always equal to the value of 'n'.

**32. Magnetic Quantum Number (m) :**

- It gives the number of permitted orientation of subshells.
- The value of m varies from  $-l$  to  $+l$  through zero.
- Degenerate orbitals – Orbitals having the same energy are known as degenerate orbitals.  
e.g. for p subshell  $P_x$ ,  $P_y$  and  $P_z$  are degenerate orbital.
- The number of degenerate orbitals of s subshell = 0.

**33. Spin quantum number (s) :**

- The value of 's' is  $+\frac{1}{2}$  or  $-\frac{1}{2}$ , which is signified as the spin or rotation or direction of electron on it's axis during the movement.
- The spin may be clockwise or anticlockwise.
- It represents the value of spin angular momentum is equal to  $\frac{h}{2\pi} \sqrt{s(s+1)}$ .
- Maximum spin of an atom =  $\frac{1}{2} \times$  number of unpaired electron.

**34. Shape and size of orbitals**

An orbital is the region of space around the nucleus within which the probability of finding an electron of given energy is maximum (90-95%). The shape of this region (electron cloud) gives the shape of the orbital. It is basically determined by the azimuthal quantum number  $l$ , while the orientation of orbital depends on the magnetic quantum number (m).

## 02 Structure of Atom

### 35. s-orbital ( $l = 0$ ):

These orbitals are spherical and symmetrical about the nucleus. The probability of finding the electron is maximum near the nucleus and keeps on decreasing as the distance from the nucleus increases. There is vacant space between two successive s-orbitals known as radial node. But there is no radial node for 1s orbital since it is starting from the nucleus.

### 36. p-orbital ( $l = 1$ ):

The probability of finding the p-electron is maximum in two lobes on the opposite sides of the nucleus. This gives rise to dumb-bell shape for the p-orbital  $l = 1$ . Hence,  $m = -1, 0, +1$ . Thus, p-orbital have three different orientations. These are designated as  $p_x, p_y$  &  $p_z$  depending upon whether the density of electron is maximum along the x y and z axis respectively.

### 37. d-orbital ( $l = 2$ ):

For d-orbitals,  $l = 2$ . Hence  $m = -2, -1, 0, +1, +2$ . Thus there are 5d orbitals. They have relatively complex geometry. Out of the five orbitals, the three ( $d_{xy}, d_{yz}, d_{zx}$ ) project in between the axis and the other two  $d_{z^2}$  and  $d_{x^2-y^2}$  lie along the axis.

### 38. Spherical nodes :

The spherical surface where probability of finding the electron is zero, is called spherical nodes.

No. of spherical nodes in any orbital =  $n-l-1$

### 39. Nodal Plane :

This is a plane passing through the nucleus where the probability of finding the electron is zero.

Number of nodal plane in a orbital =  $l$

Orbital	Nodal plane
$p_x$	yz
$p_y$	xz
$p_z$	xy
$d_{xy}$	yz,zx
$d_{yz}$	xy,xz
$d_{zx}$	xy,yz

### 40. Electronic Configuration

The distribution of electrons in different orbitals of atom is known as electronic configuration of the atoms. Filling up of orbitals in the ground state of atom is governed by the following rules:

### 41. Aufbau Principle

According to this principle, “In the ground state, the atomic orbitals are filled in order of increasing energies i.e. in the ground state the electrons first occupy the lowest energy orbitals available”.

Lower the value of  $n+l$ , lower is the energy of the orbital and such an orbital will be filled up first.

When two orbitals have same value of  $(n+l)$  the orbital having lower value of “ $n$ ” has lower energy and such an orbital will be filled up first.

Thus, order of filling up of orbitals is as follows:

$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d$

### 42. Pauli’s Exclusion Principle

According to this principle, “No two electrons in an atom can have same set of all the four quantum numbers  $n$ ,  $l$ ,  $m$  and  $s$ .”

According to this principle an orbital can accommodate at the most two electrons with spins opposite to each other. It means that an orbital can have 0, 1, or 2 electron. If an orbital has two electrons they must be of opposite spin.

### 43. Hund’s Rule of Maximum Multiplicity

According to this rule “Electron filling will not take place in orbitals of same energy until all the available orbitals of a given subshell contain one electron each with parallel spin”.

### 44. Extra stability If half filled and completely filled orbitals

#### Symmetry of orbitals

If the shift of an electron from one orbital to another orbital differing slightly in energy results in the symmetrical electronic configuration. It becomes more stable. For example  $p^3$ ,  $d^5$ ,  $f^7$  configurations are more stable than their near once.

#### Exchange energy

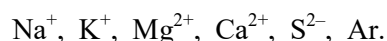
In case of half filled and completely filled orbitals, the exchange energy is maximum and is greater than the loss of energy due to the transfer of electron from a higher to a lower sublevel e.g. from 4s to 3d orbitals in case of Cu and Cr.

## CBSE Exam Pattern Exercise

### Subjective Questions (1)

**(Q. No. 1 to 2) One Mark**

1. Which of the following are isoelectronic species, i.e., those having the same number of electrons?



2. A certain particle carries  $2.5 \times 10^{-16}$  C of static electric charge. Calculate the number of electrons present in it.

**(Q. No. 3 to 4) Two Marks**

3. An element with mass number 81 contains 31.7% more neutrons as compared to protons. Assign the atomic symbol.
4. If the diameter of a carbon atom is 0.15 nm, calculate the number of carbon atoms which can be placed side by side in a straight line across the length of scale of length 20 cm long.

**(Q. No. 5 to 6) Three Marks**

5. How much energy is required to ionise a hydrogen atom if the electron occupies  $n = 5$  orbit? Compare your answer with the ionisation energy of hydrogen atom (energy required to remove the electron from  $n = 1$  orbit).
6. The quantum numbers of four electrons are given below. Arrange them in order of increasing energies. List if any of these combination (s) has/have the same energy:
- $n = 4, l = 2, m_l = -2, m_s = -1/2$
  - $n = 3, l = 2, m_l = -1, m_s = +1/2$
  - $n = 4, l = 1, m_l = 0, m_s = +1/2$
  - $n = 3, l = 2, m_l = -2, m_s = -1/2$

**(Q. No. 7) Four Marks**

7. If the position of the electron is measured within an accuracy of  $\pm 0.002$  nm, calculate the uncertainty in the momentum of the electron. Suppose the momentum of the electron is  $h/(4\pi \times 0.05)$  nm, is there any problem in defining this value?

**(Q. No. 8 to 10) Five Marks**

8. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition,  $n = 4$  to  $n = 2$  of  $\text{He}^+$  spectrum?
9. The work function for caesium atom is 1.9 eV. Calculate (a) the threshold wavelength and (b) the threshold frequency of the radiation. If the caesium element is irradiated with a wavelength

500 nm, calculate the kinetic energy and the velocity of the ejected photoelectron.

10.

- (i) Calculate the total number of electrons present in one mole of methane.
- (ii) Find (a) the total number and (b) the total mass of neutrons in 7 mg of  $^{14}\text{C}$  (Assume that the mass of neutron =  $1.675 \times 10^{-27}$  kg)
- (iii) Find (a) the total number and (b) the total mass of protons in 34 mg of  $\text{NH}_3$  at S.T.P.  
(Assume the mass of proton =  $1.6726 \times 10^{-27}$  kg)

Will the answer change if temperature and pressure are changed?





# Answer & Solution

Q1

No. of electron are:  $\text{Na}^+ = 11 - 1 = 10$ ,  $\text{K}^+ = 19 - 1 = 18$ ,  $\text{Mg}^{2+} = 12 - 2 = 10$ ,  
 $\text{Ca}^{2+} = 20 - 2 = 18$ ,  $\text{S}^{2-} = 16 + 2 = 18$ ,  $\text{Ar} = 18$ .

Hence, isoelectronic species are  $\text{N}^{3+}$  and  $\text{Mg}^{2+}$ ;  $\text{K}^+$ ,  $\text{Ca}^{2+}$ ,  $\text{S}^{2-}$  and Ar.

Q2

Charges carried by one electron =  $1.6022 \times 10^{-19} \text{C}$

$\therefore$  Electrons present in particle carrying  $2.5 \times 10^{-16} \text{C}$  charges =  $\frac{2.5 \times 10^{-16}}{1.6022 \times 10^{-19}} = 1560$

Q3

Mass number = 81, i.e.,  $p + n = 81$

If protons =  $x$ , then neutrons =  $x + \frac{31.7}{100} \times x = 1.317x$

$\therefore x + 1.317x = 81$  or  $2.317x = 81$  or  $x = \frac{81}{2.317} = 35$

Thus, protons = 35, i.e., atomic no. = 35

Hence, the symbol is  $\frac{81}{35}\text{Br}$

Q4

Diameter of carbon atom = 0.15 nm =  $0.15 \times 10^{-9} \text{ m}$   $1.5 \times 10^{-10} \text{ m}$

Length along which atoms are to be placed = 20 cm  $20 \times 10^{-2} \text{ m} = 2 \times 10^{-1} \text{ m}$

$\therefore$  No. of C-atoms which can be placed along the line =  $\frac{2 \times 10^{-1}}{1.5 \times 10^{-10}} = 1.33 \times 10^9$

Q5

$$E_n = \frac{21.8 \times 10^{-9}}{n^2} \text{ J atom}^{-1}$$

For ionization from 5<sup>th</sup> orbit,  $n_1 = 5$ ,  $n_2 = \infty$

$$\therefore \Delta E = E_2 - E_1 = -21.8 \times 10^{-19} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = 21.8 \times 10^{-19} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = 21.8 \times 10^{-19}$$

## 02 Structure of Atom

$$\left(\frac{1}{5^2} - \frac{1}{\infty}\right) = 8.72 \times 10^{-20} \text{ J}$$

For ionization from 1<sup>st</sup> orbit,  $n_1 = 1$ ,  $n_2 = \infty$

$$\Delta E' = 21.8 \times 10^{-19} \left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 21.8 \times 10^{-19} \text{ J}$$

$$\frac{\Delta E'}{\Delta E} = \frac{21.8 \times 10^{-19}}{8.72 \times 10^{-20}} = 25$$

Thus, the energy required to remove electron from 1<sup>st</sup> orbit is 25 times than that required to removed electron from 5<sup>th</sup> orbit.

Q6

The orbitals occupied by the electron are

- (i) 4 d
- (ii) 3 d
- (iii) 4 p
- (iv) 3 d
- (v) 3 p
- (vi) 4 p

Their energies will be in the order:

$$(v) < (ii) = (iv) = < (vi) = (iii) < (i)$$

Q7

$$\Delta x = 0.002 \text{ nm} = 2 \times 10^{-3} \text{ nm} = 2 \times 10^{-12} \text{ m}$$

$$\Delta x \times \Delta p = \frac{h}{4\pi} \therefore \Delta p = \frac{h}{4\pi \Delta x} = \frac{6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}}{4 \times 3.14 \times (2 \times 10^{-12} \text{ m})} = 2.638 \times 10^{-23} \text{ kg m s}^{-1}$$

$$\text{Actual momentum} = \frac{h}{4\pi \times 0.05 \text{ nm}} = \frac{h}{4\pi \times 5 \times 10^{-11} \text{ m}} = \frac{6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}}{4 \times 3.14 \times 5 \times 10^{-11} \text{ m}}$$

$$= 1.055 \times 10^{-24} \text{ kg m s}^{-1}$$

It cannot be defined as the actual magnitude of the momentum is smaller than the uncertainty.

Q8

$$\text{For H-like particles in general } \bar{V} = \frac{2\pi^2 m Z^2 e^4}{ch^3} \left( \frac{1}{n_1} - \frac{1}{n_2} \right) = RZ^2 \left( \frac{1}{n_1} - \frac{1}{n_2} \right)$$

$\therefore$  For He<sup>+</sup> spectrum, for Balmer transition,  $n = 4$  to  $n = 2$ .

$$\bar{V} = \frac{1}{\lambda} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right) = R \times 4 \times \frac{3}{16} = \frac{3R}{4}$$

$$\text{For hydrogen spectrum } \bar{V} = \frac{1}{\lambda} = R \left( \frac{1}{n_1} - \frac{1}{n_2} \right) = \frac{3}{4} R \text{ or } \frac{1}{n_1} - \frac{1}{n_2} = \frac{3}{4}$$

which can be so far  $n_1 = 1$  and  $n_2 = 2$ . i.e., the transition is from  $n = 2$  to  $n = 1$

Q9

(i) Work function ( $W_0$ ) =  $h \nu_0$

$$\therefore \nu_0 = \frac{W_0}{h} = \frac{1.9 \times 1.602 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J s}} = 4.59 \times 10^{14} \text{ s}^{-1} \quad (1 \text{ eV} = 1.602 \times 10^{-19} \text{ J})$$

## 02 Structure of Atom

$$(ii) \quad \therefore \lambda_0 = \frac{c}{v_0} = \frac{3.0 \times 10^8 \text{ ms}^{-1}}{6.59 \times 10^{14} \text{ s}^{-1}} = 6.54 \times 10^{-7} \text{ m} = 654 \times 10^{-9} \text{ m} = \mathbf{654 \text{ nm}}$$

$$(iii) \quad \text{K.E. of ejected electron} = h(v - v_0) = hc \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$= (6.626 \times 10^{-34} \text{ J s}) (3.0 \times 10^8 \text{ m s}^{-1}) \left( \frac{1}{500 \times 10^{-9} \text{ m}} - \frac{1}{654 \times 10^{-9} \text{ m}} \right)$$

$$= \frac{6.626 \times 3.0 \times 10^{-26}}{10^{-9}} \left( \frac{154}{500 \times 654} \right) \text{ J} = \mathbf{9.36 \times 10^{-20} \text{ J}}$$

$$\text{K.E.} = \frac{1}{2} m v^2 = 9.36 \times 10^{-20} \text{ J or kg m}^2 \text{ s}^{-2}$$

$$\therefore \frac{1}{2} \times (9.11 \times 10^{-31} \text{ kg}) v^2 = 9.36 \times 10^{-20} \text{ kg m}^2 \text{ s}^{-2}$$

$$\text{or } v^2 = 2.055 \times 10^{11} \text{ m}^2 \text{ s}^{-2} = 20.55 \times 10^{10} \text{ m}^2 \text{ s}^{-2} \quad \text{or } v = \mathbf{4.53 \times 10^5 \text{ ms}^{-1}}.$$

Q10

(i) 1 molecule of  $\text{CH}_4$  contains electrons = 6+4=10

$\therefore$  1 mole, i.e.,  $6.022 \times 10^{23}$  molecules will contain electrons =  $\mathbf{6.022 \times 10^{23}}$

(ii)

(a) 1 g atom of  $^{14}\text{C}$  = 14 g =  $6.022 \times 10^{23}$  atoms =  $(6.022 \times 10^{23}) \times 8$  neutrons.)  
(as each  $^{14}\text{C}$  atom has 14-6=8 neutrons)

Thus, 14 g or 14000 mg have  $8 \times 6.022 \times 10^{23}$  neutrons

$$\therefore 7 \text{ mg will have neutron} = \frac{8 \times 6.022 \times 10^{23}}{14000} \times 7 = \mathbf{2.4088 \times 10^{21}}$$

(b) Mass of 1 neutrons =  $1.675 \times 10^{-27}$  kg

$$\therefore \text{Mass of } 2.4088 \times 10^{21} \text{ neutrons} = (2.4088 \times 10^{21}) (1.675 \times 10^{-27} \text{ kg}) = \mathbf{4.0347 \times 10^{-6} \text{ kg}}$$

(iii)

(a) 1 mol of  $\text{NH}_3$  = 17 g  $\text{NH}_3$  =  $6.022 \times 10^{23}$  molecules of  $\text{NH}_3$   
=  $(6.022 \times 10^{23}) \times (7+3)$  protons =  $6.022 \times 10^{24}$  protons

$$\therefore 34 \text{ mg, i.e., } 0.034 \text{ g } \text{NH}_3 = \frac{6.022 \times 10^{24}}{17} \times 0.034 = \mathbf{1.2044 \times 10^{22} \text{ protons.}}$$

(b) Mass of one proton =  $1.6726 \times 10^{-27}$  kg

$$\therefore \text{Mass of } 1.2044 \times 10^{22} \text{ protons} = (1.6726 \times 10^{-27}) \times (1.2044 \times 10^{22}) \text{ kg} = \mathbf{2.0145 \times 10^{-5} \text{ kg}}$$

There is no effect of temperature and pressure.



CLASS 11

# CBSE

# MATHEMATICS

- ▶ Specially designed eBook for complete CBSE syllabus
- ▶ CBSE preparation strategy & direction, speedy revision
- ▶ Chapter-wise important Problem-Solving
- ▶ Theory, Formulas & Diagrams to crack CBSE
- ▶ Updated New Pattern Objective questions
- ▶ Detailed high quality video lectures by experienced faculties
- ▶ CBSE full preparation for Class XI & XII



**misostudy**  
connect smart learning

**JEE·NEET·AIIMS·CBSE·FOUNDATION**  
[www.misostudy.com](http://www.misostudy.com)

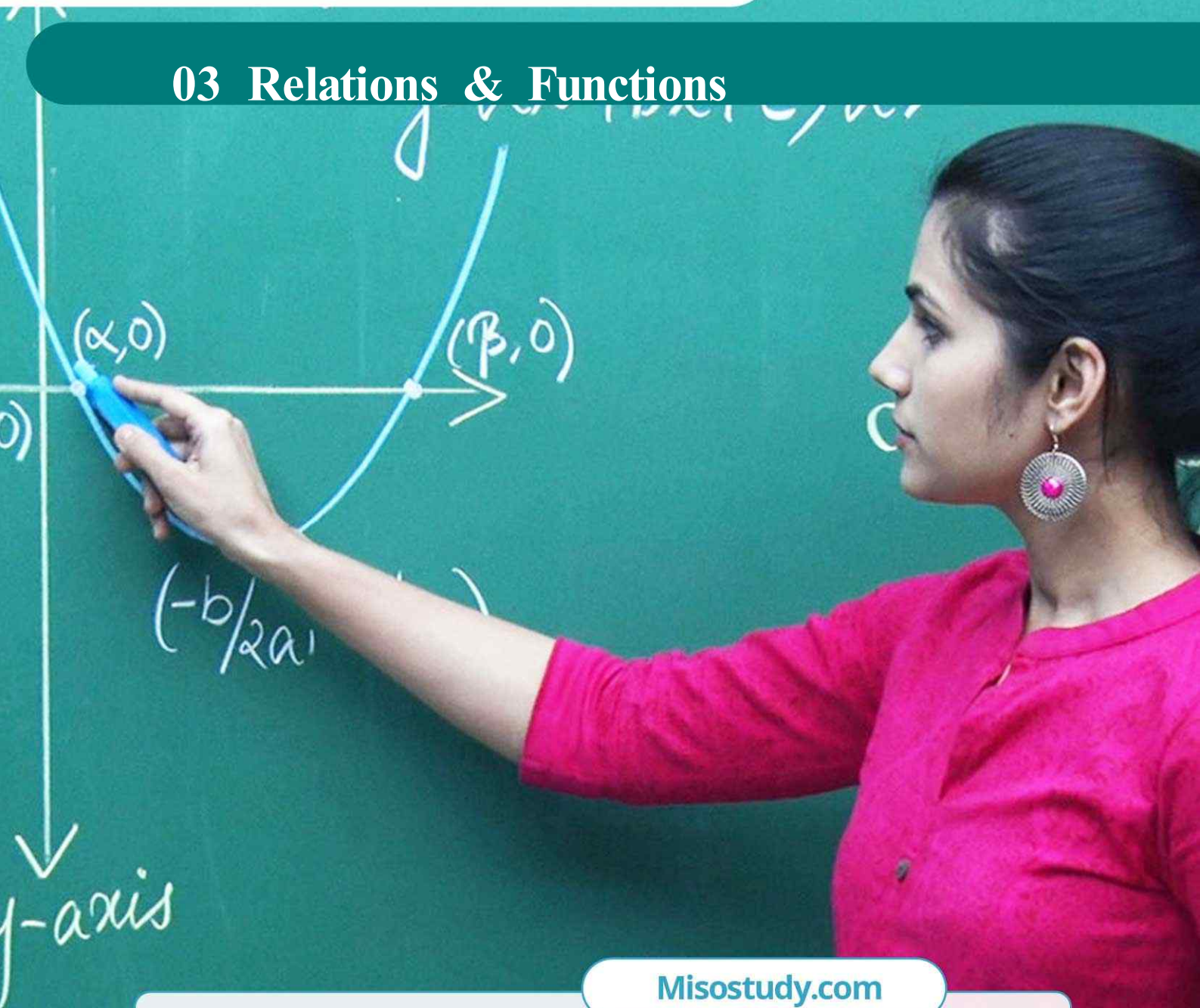
☎ 8929803804 (MON-FRI: 9am-6pm) [support@misostudy.com](mailto:support@misostudy.com)

MISO STUDY INDIA PVT. LTD.  
2ND FLOOR 65-A, OMPRO TOWER, KALU SARAI, NEW DELHI, DELHI 110016

# Geometric Representation

## Class 11 | Mathematics

### 03 Relations & Functions



Misostudy.com



#### Scientific eBook

This is a sample eBook. The eBook provides chapter-wise theories and examples, formulas, diagrams, and Exam pattern Problem-Solving of the full syllabus.



#### Complete video lectures

The theory and Problem-Solving in this eBook provide specially designed the video lectures by experienced faculties. Get Free trial and preview on Misostudy.com.

## 01. Definition

### (i) ORDERED PAIR

An ordered pair consists of two objects or elements in a given fixed order.

For example, if  $A$  and  $B$  are any two sets, then by an ordered pair of elements we mean pair  $(a, b)$  in that order, where  $a \in A, b \in B$ .

**NOTE** 

An ordered pair is not a set consisting of two elements. The ordering of the two elements in an ordered pair is important and the two elements need not be distinct.

### (ii) EQUALITY OF ORDERED PAIRS

Two ordered pairs  $(a_1, b_1)$  and  $(a_2, b_2)$  are equal iff

$$a_1 = a_2 \text{ and } b_1 = b_2$$

i.e.,  $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2 \text{ and } b_1 = b_2$

**Example** Find the values of  $a$  and  $b$ , if  $(3a - 2, b + 3) = (2a - 1, 3)$ .

**Solution** By the definition of equality of ordered pairs, we have

$$(3a - 2, b + 3) = (2a - 1, 3)$$

$$\Leftrightarrow 3a - 2 = 2a - 1 \text{ and } b + 3 = 3$$

$$\Leftrightarrow a = 1 \text{ and } b = 0$$

### (iii) CARTESIAN PRODUCT OF SETS

Let  $A$  and  $B$  be any two non-empty sets. The set of all ordered pairs  $(a, b)$  such that  $a \in A$  and  $b \in B$  is called the cartesian product of the sets  $A$  and  $B$  and is denoted by  $A \times B$ .

Thus,  $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If  $A = \phi$  or  $B = \phi$ , then we define  $A \times B = \phi$

**Example** If  $A = \{2, 4, 6\}$  and  $B = \{1, 2\}$ , then

$$A \times B = \{2, 4, 6\} \times \{1, 2\} = \{(2, 1), (2, 2), (4, 1), (4, 2), (6, 1), (6, 2)\}$$

$$\text{and, } B \times A = \{1, 2\} \times \{2, 4, 6\} = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6)\}$$

## 02. Number of Elements in the Cartesian Product of Two Sets

### RESULT

If  $A$  and  $B$  are two finite sets, then  $n(A \times B) = n(A) \times n(B)$ .

**PROOF**

Let  $A = \{a_1, a_2, a_3, \dots, a_m\}$  and  $B = \{b_1, b_2, b_3, \dots, b_n\}$  be two sets having  $m$  and  $n$  elements respectively. Then,

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), \dots, (a_1, b_n) \\ (a_2, b_1), (a_2, b_2), (a_2, b_3), \dots, (a_2, b_n) \\ \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \\ (a_m, b_1), (a_m, b_2), (a_m, b_3), \dots, (a_m, b_n)\}$$

Clearly, in the tabular representation of  $A \times B$  there are  $m$  rows of ordered pairs and each row has  $n$  distinct ordered pairs.

So,  $A \times B$  has  $mn$  elements.

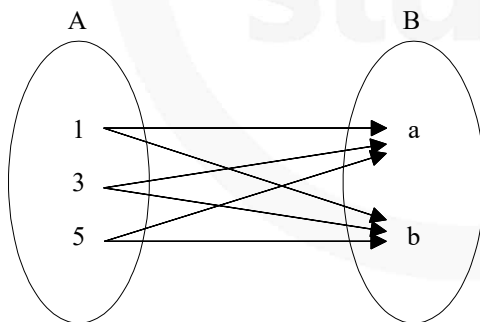
Hence,  $n(A \times B) = mn = n(A) \times n(B)$

- REMARK** (i) If either  $A$  or  $B$  is an infinite set, then  $A \times B$  is an infinite set.  
(ii) If  $A, B, C$  are finite sets, then  $n(A \times B \times C) = n(A) \times n(B) \times n(C)$

**03. Diagrammatic Representation of Cartesian Product of Two Sets**

In order to represent  $A \times B$  by an arrow diagram, we first draw Venn diagrams representing sets  $A$  and  $B$  one opposite to the other as shown in Figure. Now, we draw line segments starting from each element of  $A$  and terminating to each element of set  $B$ .

If  $A = \{1, 2, 3\}$  and  $B = \{a, b\}$ , then following figure gives the arrow diagram of  $A \times B$ .

**04. Some Useful Results**

**RESULT 1**

For any three sets  $A, B, C$ , prove that:

(i)  $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ .

**Proof (i)** Let  $(a, b)$  be an arbitrary element of  $A \times (B \cup C)$ . Then,  
 $(a, b) \in A \times (B \cup C) \Rightarrow a \in A$  and  $b \in B \cup C$  [by def.]  
 $\Rightarrow a \in A$  and  $(b \in B$  or  $b \in C)$  [by def. of union]  
 $\Rightarrow (a \in A$  and  $b \in B)$  or  $(a \in A$  and  $b \in C)$   
 $\Rightarrow (a, b) \in A \times B$  or  $(a, b) \in A \times C \Rightarrow (a, b) \in (A \times B) \cup (A \times C)$   
 $\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$  ... (i)

Again, let  $(x, y)$  be an arbitrary element of  $(A \times B) \cup (A \times C)$ . Then,  
 $(x, y) \in (A \times B) \cup (A \times C) \Rightarrow (x, y) \in A \times B$  or  $(x, y) \in A \times C$   
 $\Rightarrow (x \in A$  and  $y \in B)$  or  $(x \in A$  and  $y \in C)$   
 $\Rightarrow x \in A$  ( $y \in B$  or  $y \in C$ )  
 $\Rightarrow x \in A$  and  $y \in (B \cup C) \Rightarrow (x, y) \in A \times (B \cup C)$   
 $\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$  ... (ii)

Hence, from (i) and (ii), we have  $A \times (B \cup C)$ . Then,

**Proof (ii)** Let  $(a, b)$  be an arbitrary element of  $A \times (B \cap C)$ . Then,  
 $(a, b) \in A \times (B \cap C) \Rightarrow a \in A$  and  $b \in (B \cap C)$  [by def.]  
 $\Rightarrow a \in A$  and  $(b \in B$  and  $b \in C)$   
 $\Rightarrow (a \in A$  and  $b \in B)$  and  $(a \in A$  and  $b \in C)$   
 $\Rightarrow (a, b) \in A \times B$  and  $(a, b) \in A \times C$  [by def.]  
 $\Rightarrow (a, b) \in (A \times B) \cap (A \times C)$   
 $\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$  ... (i)

Again, let  $(x, y)$  be an arbitrary element of  $(A \times B) \cap (A \times C)$ . Then,  
 $(x, y) \in (A \times B) \cap (A \times C) \Rightarrow (x, y) \in (A \times B)$  and  $(x, y) \in (A \times C)$   
 $\Rightarrow (x \in A$  and  $y \in B)$  and  $(x \in A$  and  $y \in C)$   
 $\Rightarrow x \in A$  and  $(y \in B$  or  $y \in C)$   
 $\Rightarrow x \in A$  and  $y \in (B \cap C) \Rightarrow (x, y) \in A \times (B \cap C)$   
 $\therefore (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$  ... (ii)

Hence, from (i) and (ii), we get

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

**RESULT 2**

For any three sets  $A, B, C$ , prove that:

$$A \times (B - C) = (A \times B) - (A \times C)$$

**Proof** Let  $(a, b)$  be an arbitrary element of  $A \times (B - C)$ . Then,  
 $(a, b) \in A \times (B - C) \Rightarrow a \in A$  and  $b \in (B - C) \Rightarrow a \in A$  and  
 $(b \in B$  and  $b \notin C)$   
 $\Rightarrow (a \in A$  and  $b \in B)$  and  $(a \in A$  and  $b \notin C)$   
 $\Rightarrow (a, b) \in (A \times B)$  and  $(a, b) \notin (A \times C) \Rightarrow (a, b) \in (A \times B) - (A \times C)$   
 $\therefore A \times (B - C) \subseteq (A \times B) - (A \times C)$  ... (i)

Again, let  $(x, y)$  be an arbitrary element of  $(A \times B) - (A \times C)$ . Then,  
 $(x, y) \in (A \times B) - (A \times C) \Rightarrow (x, y) \in A \times B$  and  $(x, y) \notin A \times C$   
 $\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \notin C) \Rightarrow x \in A$  and  
 $(y \in B \text{ and } y \notin C)$   
 $\Rightarrow x \in A \text{ and } y \in (B - C) \Rightarrow (x, y) \in A \times (B - C)$   
 $\therefore (A \times B) - (A \times C) \subseteq A \times (B - C)$  ... (ii)  
Hence, from (i) and (ii), we get  
 $A \times (B - C) = (A \times B) - (A \times C)$

**RESULT 7**

For any sets  $A, B, C, D$  prove that:

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

**Proof** Let  $(a, b)$  be an arbitrary element of  $(A \times B) \cap (C \times D)$ . Then,

$(a, b) \in (A \times B) \cap (C \times D) \Rightarrow (a, b) \in A \times B$  and  $(a, b) \in C \times D$   
 $\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in C \text{ and } b \in D)$   
 $\Rightarrow (a \in A \text{ and } b \in C) \text{ and } (a \in B \text{ and } b \in D)$   
 $\Rightarrow a \in (A \cap C) \text{ and } b \in (B \cap D) \Rightarrow (a, b) \in (A \cap C) \times (B \cap D)$   
 $\therefore (A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$

Similarly,  $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$

Hence,  $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

**Corollary** For any sets  $A$  and  $B$ , prove that  $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$ .

**06. Concept of Relations****(i) RELATION**

Let  $A$  and  $B$  be two sets. Then a relation  $R$  from  $A$  to  $B$  is a subset of  $A \times B$ . Thus,  $R$  is a relation from  $A$  to  $B \Leftrightarrow R \subseteq A \times B$ .

If  $R$  is a relation from a non-void set  $A$  to a non-void set  $B$  and if  $(a, b) \in R$ , then we write  $aRb$  which is read as ' $a$  is related to  $b$  by the relation  $R$ '. If  $(a, b) \notin R$ , then we write  $a \not R b$  and we say that  $a$  is not related to  $b$  by the relation  $R$ .

**(ii) TOTAL NUMBER OF RELATIONS**

Let  $A$  and  $B$  be two non-empty finite sets consisting of  $m$  and  $n$  elements respectively. Then  $A \times B$  consists of  $mn$  ordered pairs. So, total number of subsets of  $A \times B$  is  $2^{mn}$ . Since each subset of  $A \times B$  defines a relation from  $A$  to  $B$ , so total numbers of relations from  $A$  to  $B$  is  $2^{mn}$ . Among these  $2^{mn}$  relations the void relation  $\phi$  and the universal relation  $A \times B$  are trivial relations from  $A$  to  $B$ .

## 07. Representation of a Relation

A relation from a set  $A$  to a set  $B$  can be represented in any one of the following forms:

### (i) ROSTER FORM

In this form a relation is represented by the set of all ordered pairs belonging to  $R$ .

**Example** If  $R$  is a relation from set  $A = \{-2, -1, 0, 1, 2\}$  to set  $B = \{0, 1, 4, 9, 10\}$  by the rule

$$a R b \Leftrightarrow a^2 = b$$

Then,  $0 R 0$ ,  $-2 R 4$ ,  $-1 R 1$ ,  $1 R 1$  and  $2 R 4$ .

So,  $R$  can be described in Roster form as follows:

$$R = \{(0, 0), (-1, 1), (-2, 4), (1, 1), (2, 4)\}$$

### (ii) SET-BUILDER FORM

In this form the relation  $R$  from set  $A$  to set  $B$  is represented as

$$R = \{(a, b) : a \in A, b \in B \text{ and } a, b \text{ satisfy the rule which associates } a \text{ and } b\}$$

**Example** If  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots\right\}$  and  $R$  is a relation from  $A$  to  $B$  given by

$$R = \left\{ \left(1, 1\right), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right) \right\}$$

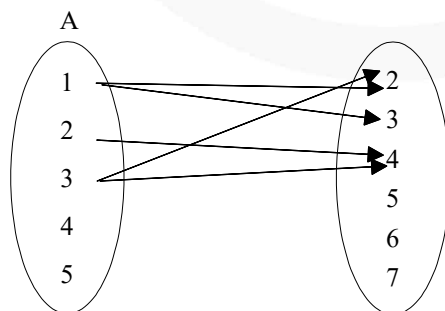
Then,  $R$  in set-builder form can be described as follows:

$$R = \left\{ (a, b) : a \in A, b \in B \text{ and } b = \frac{1}{a} \right\}$$

### (iii) BY ARROW DIAGRAM

In order to represent a relation from set  $A$  to a set  $B$  by an arrow diagram, we draw arrows from first components to the second components of all ordered pairs belonging to  $R$ .

**Example** Relation  $R = \{(1, 2), (2, 4), (3, 2), (1, 3), (3, 4)\}$  from set  $A = \{1, 2, 3, 4, 5\}$  to set  $B = \{2, 3, 4, 5, 6, 7\}$  can be represented by the following arrow diagram:



## 08. Domain and Range of a Relation

Let  $R$  be a relation from a set  $A$  to a set  $B$ . Then the set of all first components or coordinates of the ordered pairs belonging to  $R$  is called the domain or  $R$ , while the set of all second components or coordinates of the ordered pairs in  $R$  is called the range or  $R$ .

Thus,  $\boxed{\text{Dom } (R) = \{a : (a, b) \in R\} \text{ and Range } (R) = \{b : (a, b) \in R\}}$ .

★ **RELATION ON A SET** Let  $A$  be a non-void set. Then, a relation from  $A$  to itself i.e. a subset of  $A \times A$ , is called a relation on set  $A$ .

**Example** If  $A = \{1, 3, 5, 7\}$ ,  $B = \{2, 4, 6, 8, 10\}$  and let  $R = \{(1, 8), (3, 6), (5, 2), (1, 4)\}$  be a relation from  $A$  to  $B$ . Then,

$$\text{Dom } (R) = \{1, 3, 5\} \text{ and Range } (R) = \{8, 6, 2, 4\}$$

## 10. Function as a Special Kind of Relation

### DEFINITION

Let  $A$  and  $B$  be two non-empty sets. A relation  $f$  from  $A$  to  $B$ , i.e., a sub-set of  $A \times B$ , is called a function (or a mapping or a map) from  $A$  to  $B$ , if

- (i) for each  $a \in A$  there exists  $b \in B$  such that  $(a, b) \in f$
- (ii)  $(a, b) \in f$  and  $(a, c) \in f \Rightarrow b = c$ .

## 11. Function as a Correspondence

### DEFINITION

Let  $A$  and  $B$  be two non-empty sets. Then a function ' $f$ ' from set  $A$  to set  $B$  is a rule or method or correspondence which associates elements of set  $A$  to elements of set  $B$  such that:

- (i) all elements of set  $A$  are associated to elements in set  $B$ .
- (ii) an element of set  $A$  is associated to a unique element in set  $B$ .

In other words, a function ' $f$ ' from a set  $A$  to a set  $B$  associates each element of set  $A$  to a unique element of set  $B$ .

### NOTE

If an element  $a \in A$  is associated to an element  $b \in B$ , then  $b$  is called 'the  $f$ -image of  $a$ ' or 'image of  $a$  under  $f$ ' or 'the value of the function  $f$  at  $a$ '. Also,  $a$  is called the pre-image of  $b$  under the function  $f$ . We write it as  $b = f(a)$



## 12. Description of a Function

Let  $f: A \rightarrow B$  be a function such that the set  $A$  consists of a finite number of elements. Then,  $f(x)$  be described by listing the values which it attains at different points of its domain. For example, if  $A = \{-1, 1, 2, 3\}$  and  $B$  is the set of real numbers, then a function  $f: A \rightarrow B$  can be described as  $f(-1) = 3, f(1) = 0, f(2) = 3/2$  and  $f(3) = 0$ . In case,  $A$  is an infinite set, then  $f$  cannot be described by listing the image at points in its domain. In such cases functions are generally described by a formula. For example,  $f: Z \rightarrow Z$  given by  $f(x) = x^2 + 1$  or  $f: R \rightarrow R$  given by  $f(x) = e^x$  etc.

## 13. Domain, Co-Domain and Range of a Function

Let  $f: A \rightarrow B$ . Then, the set  $A$  is known as the domain of  $f$  and the set  $B$  is known as the co-domain of  $f$ . The set of all  $f$ -images of elements of  $A$  is known as the range of  $f$  or image set of  $A$  under  $f$  and is denoted by  $f(A)$ .

Thus,  $f(A) = \{f(x) : x \in A\} = \text{Range of } f$

Clearly,  $f(A) \subseteq B$ .

## 14. Equal Function

### DEFINITION

Two functions  $f$  and  $g$  are said to be equal iff

- (i) domain of  $f =$  domain of  $g$ ,
  - (ii) co-domain of  $f =$  co-domain of  $g$ ,
- and (iii)  $f(x) = g(x)$  for every  $x$  belonging to their common domain.

If two function  $f$  and  $g$  are equal, then we write  $f = g$ .

**Example** Let  $A = \{1, 2\}$ ,  $B = \{3, 6\}$  and  $f: A \rightarrow B$  given by  $f(x) = x^2 + 2$  and  $g: A \rightarrow B$  given by  $g(x) = 3x$ . Then, we observe that  $f$  and  $g$  have the same domain and co-domain. also we have,  $f(1) = 3 = g(1)$  and  $f(2) = 6 = g(2)$   
Hence,  $f = g$ .

## 15. Real Valued Function

A function  $f: A \rightarrow B$  is called a real valued function, if  $B$  is a subset of  $R$  (set of all real numbers).

If  $A$  and  $B$  both are subsets of  $R$ , then  $f$  is called a real function.

### NOTE

In practice, real functions are described by giving the general expression or formula describing it without mentioning its domain and co-domain.

## 16. Domain of Real Functions

The domain of  $f(x)$  is the set of all those real numbers for which  $f(x)$  is meaningful.

## 17. Range of Real Functions

The range of a real function of a real variable is the set of all real values taken by  $f(x)$  at points in its domain. In order to find the range of a real function  $f(x)$ , we may use the following algorithm.

### ALGORITHM

**STEP I** Put  $y = f(x)$

**STEP II** Solve the equation  $y = f(x)$  for  $x$  in terms of  $y$ . Let  $x = \phi(y)$ .

**STEP III** Find the values of  $y$  for which the values of  $x$ , obtained from  $x = \phi(y)$ , are real and in the domain of  $f$ .

**STEP IV** The set of values of  $y$  obtained in step *III* is the range of  $f$ .

**Example** Find the domain and range of the function  $f(x)$  given by

$$f(x) = \frac{x-2}{3-x}$$

**Solution** We have,

$$f(x) = \frac{x-2}{3-x}$$

**Domain of  $f$ :** Clearly,  $f(x)$  is defined for all  $x$  satisfying  $3-x \neq 0$  i.e.  $x \neq 3$ .

Hence, Domain ( $f$ ) =  $\mathbb{R} - \{3\}$ .

**Range of  $f$ :** Let  $y = f(x)$ , i.e.

$$y = \frac{x-2}{3-x}$$

$$\Rightarrow 3y - xy = x - 2$$

$$\Rightarrow x(y+1) = 3y+2$$

$$\Rightarrow x = \frac{3y+2}{y+1}$$

Clearly,  $x$  assumes real values for all  $y$  except  $y+1=0$  i.e.  $y=-1$ .

Hence, Range ( $f$ ) =  $\mathbb{R} - \{-1\}$ .

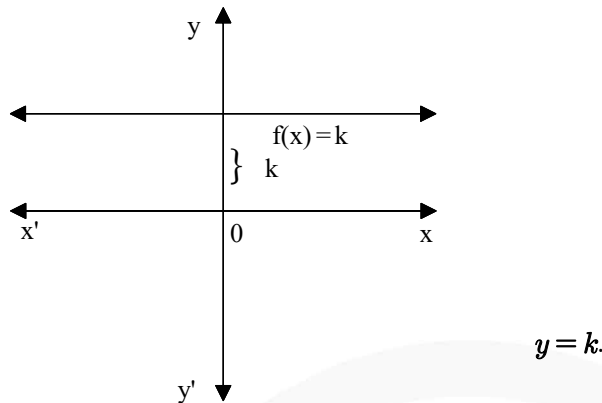
## 18. Some Standard Real Functions and Their Graphs

### CONSTANT FUNCTION

If  $k$  is a fixed real number, then a function  $f(x)$  given by

$$f(x) = k \text{ for all } x \in R$$

is called a constant function.



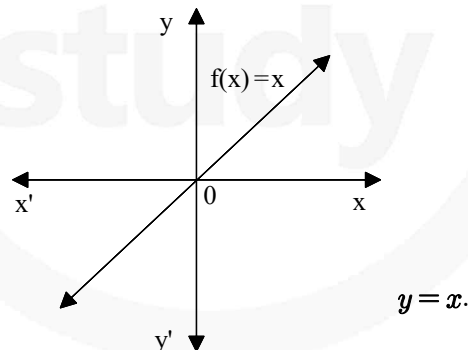
### IDENTITY FUNCTION

The function that associates each real number to itself is called the identity function and is usually denoted by  $I$ .

Thus, the function  $I: R \rightarrow R$  defined by

$$I(x) = x \text{ for all } x \in R$$

is called the identity function.



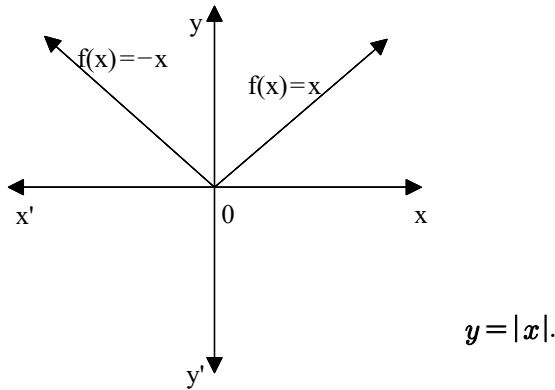
### MODULUS FUNCTION

The function  $f(x)$  defined by

$$f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

is called the modulus function.

It is also called the absolute value function.



### GREATEST INTEGER FUNCTION (FLOOR FUNCTION)

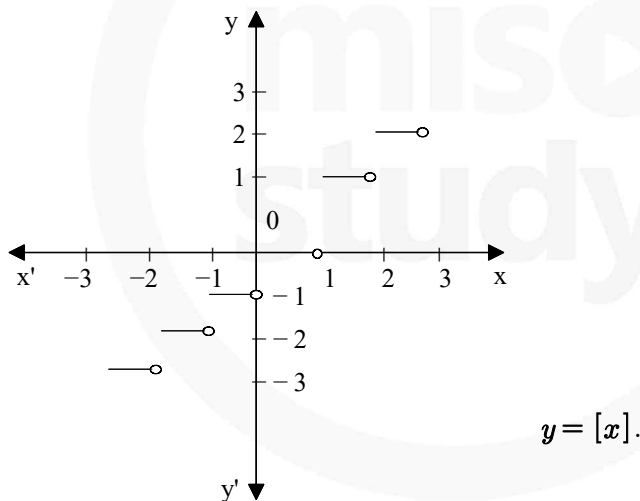
For any real number  $x$ , we use the symbol  $[x]$  or  $\lfloor x \rfloor$  to denote the greatest integer less than or equal to  $x$ .

The function  $f: R \rightarrow R$  defined by

$$f(x) = [x] \text{ for all } x \in R$$

is called the greatest integer function or the floor function.

It is also called a step function.



### SIGNUM FUNCTION

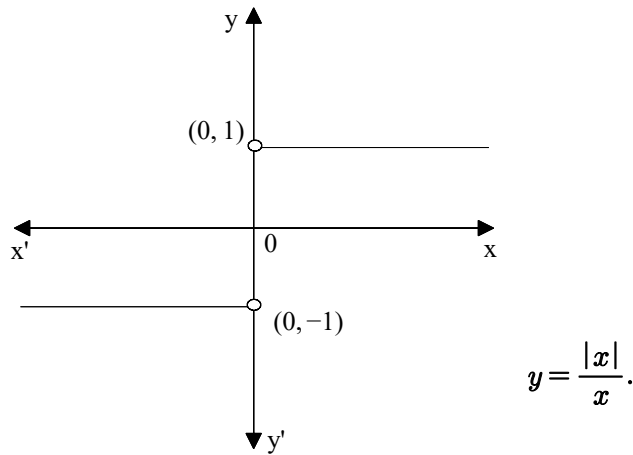
The function  $f$  defined by

$$f(x) = \begin{cases} |x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

or,

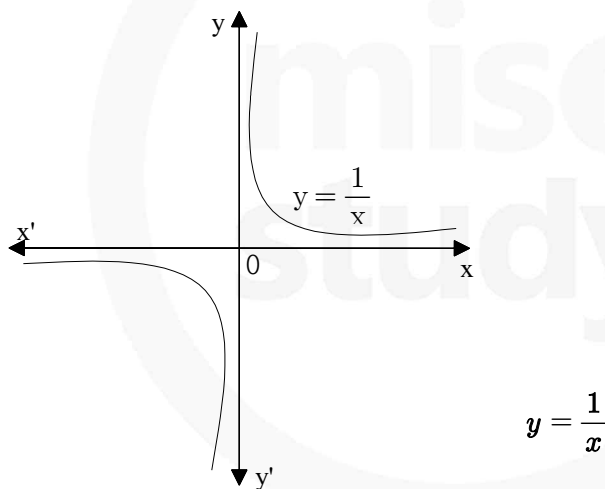
$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is called the signum function.



### RECIPROCAL FUNCTION

The function  $f: R - \{0\} \rightarrow R$  defined by  $f(x) = \frac{1}{x}$  is called the reciprocal function.

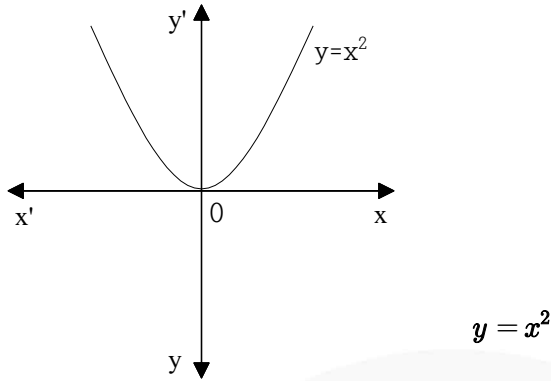


### SQUARE FUNCTION

The function  $f: R \rightarrow R$  defined by

$$f(x) = x^2$$

is called the square function.

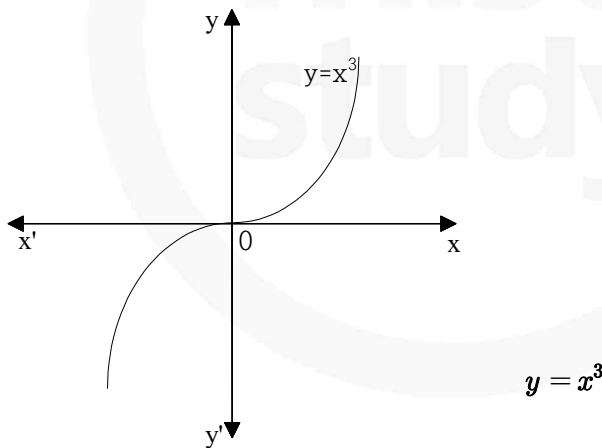


### CUBE FUNCTION

The function  $f: R \rightarrow R$  defined by

$$f(x) = x^3$$

is called the cube function.



**REMARK** (1) A function  $f: R \rightarrow R$  is said to be a polynomial function if  $f(x)$  is a polynomial in  $x$ . For example,  $f(x) = x^2 - x + 4$ ,  $g(x) = x^3 + 3x^2 + \sqrt{2}x - 1$  etc are polynomial functions.

(2) A function of the form  $f(x) = \frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are polynomials and  $q(x) \neq 0$ , is called a rational function. The domain of a rational function  $f(x) = \frac{p(x)}{q(x)}$  is the set of all real numbers, except points where  $q(x) = 0$ .

## 19. REAL FUNCTIONS

### OPERATIONS ON REAL FUNCTIONS-

In this section, we shall introduce various operations namely addition, subtraction, multiplication, division etc. on real function.

#### ADDITION

Let  $f: D_1 \rightarrow R$  and  $g: D_2 \rightarrow R$  be two real functions. Then, their sum  $f+g$  is defined as that function from  $D_1 \cap D_2$  to  $R$  which associates each  $x \in D_1 \cap D_2$  to be number  $f(x)+g(x)$ .

#### PRODUCT

Let  $f: D_1 \rightarrow R$  and  $g: D_2 \rightarrow R$  be two real functions. Then, their product ( or pointwise multiplication )  $fg$  is a function from  $D_1 \cap D_2$  to  $R$  and is defined as

$$(fg)(x) = f(x)g(x) \text{ for all } x \in D_1 \cap D_2$$

#### DIFFERENCE (SUBTRACTION)

Let  $f: D_1 \rightarrow R$  and  $g: D_2 \rightarrow R$  be two real functions. Then the difference of  $g$  from  $f$  is denoted by  $f-g$  and is defined as

$$(f-g)(x) = f(x) - g(x) \text{ for all } x \in D_1 \cap D_2$$

#### QUOTIENT

Let  $f: D_1 \rightarrow R$  and  $g: D_2 \rightarrow R$  be two real functions. Then the quotient of  $f$  by  $g$  is denoted by  $\frac{f}{g}$  and it is a function from  $D_1 \cap D_2 - \{x : g(x) = 0\}$  to  $R$  defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for all } x \in D_1 \cap D_2 - \{x : g(x) = 0\}$$

#### MULTIPLICATION OF A FUNCTION BY A SCALAR

Let  $f: D \rightarrow R$  be a real function and  $\alpha$  be a scalar (real number). Then the product  $\alpha f$  is a function from  $D$  to  $R$  and is defined as

$$(\alpha f)(x) = \alpha f(x) \text{ for all } x \in D.$$

#### RECIPROCAL OF A FUNCTION

If  $f: D \rightarrow R$  is a real function, then its reciprocal function  $\frac{1}{f}$  is a function from  $D - \{x : f(x) = 0\}$  to  $R$  and is defined as

$$\text{Example I } \left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$$

**Example I** Let  $f(x) = x^2$  and  $g(x) = 2x + 1$  be two real functions defined over the set of non-negative real numbers. Find  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(fg)(x)$  and  $\left(\frac{f}{g}\right)(x)$ .

**Solution** We have,

$$(f + g)(x) = x^2 + 2x + 1, (f - g)(x) = x^2 - 2x - 1,$$

$$(fg)(x) = x^2(2x + 1) = 2x^3 + x^2, \left(\frac{f}{g}\right)(x) = \frac{x^2}{2x + 1}, x \neq -\frac{1}{2}$$

**REMARK** (1) The sum, difference product and quotient are defined for real functions only on their common domain. These operations do not make any sense for general functions even if their domains are same, because the sum, difference, product and quotient may or may not be meaningful for the elements in their common domain.

(2) For any real function  $f: D \rightarrow R$  and  $n \in N$ , we define

$$\underbrace{(f f f \dots f)}_{n\text{-times}}(x) = \underbrace{f(x) f(x) \dots f(x)}_{n\text{-times}} = \{f(x)\}^n \text{ for all } x \in D$$

miso  
study



## CBSE Exam Pattern Exercise

### Subjective Questions (1)

**(Q 1 to 2) One Mark**

- Let  $f : R \rightarrow R$  be given by  $f(x) = x^2 + 3$ . Find the pre-images of 39 and 2 under  $f$ .
- If  $f(x) = 3x^3 - 5x^2 + 9$ , find  $f(x - 1)$ .

**(Q 3 to 4) Two Mark**

- Let  $A = \{1, 2, 3\}$  and  $B = \{x : x \in N, x \text{ is prime less than } 5\}$ . Find  $A \times B$  and  $B \times A$  &  $((A \times B) \cap (B \times A))$
- Is  $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$  a function? If this is described by the formula,  $g(x) = \alpha x + \beta$ , then what values should be assigned to  $\alpha$  and  $\beta$ ?

**(Q 5 to 7) Four Marks**

- Let  $A = \{1, 2, 3, 4, 5, 6\}$ . Define a relation  $R$  on set  $A$  by  $R = \{(x, y) : y = x + 1\}$ 
  - Depict this relation using an arrow diagram.
  - Write down the domain, co-domain and range of  $R$ .
- Find the domain and range of each of the following function:  $f(x) = \frac{3}{2-x^2}$ .
- Let  $A$  be a non-empty set such that  $A \times B = A \times C$ . Show that  $B = C$ .

**(Q 8 to 10) Four Marks**

- Let  $f$  and  $g$  be real functions defined by  $f(x) = \sqrt{x+2}$  and  $g(x) = \sqrt{4-x^2}$ . Then, find each of the following functions:
  - $f + g$
  - $f - g$
  - $fg$
  - $\frac{f}{g}$
  - $ff$
  - $gg$
- The function  $f$  is defined by  $f(x) = \begin{cases} 1-x, & x < 0 \\ 1, & x = 0 \\ x+1, & x > 0 \end{cases}$  Draw the graph of  $f(x)$ .
- Define the function  $f : R \rightarrow R$  by  $y = f(x) = x^2$ ,  $x \in R$ . Complete the Table given below by using this definition. What is the domain and range of this function? Draw the graph of  $f$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$y = f(x) = x^2$									



# Answer & Solution

Q1.

Let  $x$  be the pre-image of 39. Then,

$$f(x) = 39 \Rightarrow x^2 + 3 = 39 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$$

So, pre-images of 39 are  $-6$  and  $6$ .Let  $x$  be the pre-image of 2. Then,

$$f(x) = 2 \Rightarrow x^2 + 3 = 2 \Rightarrow x^2 = -6$$

Q2.

$$3x^3 - 5x^2 + 9$$

$$3(x-1)^3 - 5(x-1)^2 + 9$$

$$3(x^3 - 1 - 3x(x-1)) - 5(x^2 + 1 - 2x) + 9$$

$$3(x^3 - 1 - 3x^2 + 3x) - 5x^2 - 5 + 10x + 9$$

$$3x^3 - 3 - 9x^2 + 9x - 5x^2 - 5 + 10x + 9$$

$$3x^3 - 14x^2 + 19x + 1$$

Q3.

We have,

$$A = \{1, 2, 3\}$$

and,  $B = \{x : x \in N, x \text{ is prime less than } 5\} = \{2, 3\}$ 

$$\therefore A \times B = \{1, 2, 3\} \times \{2, 3\} = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

and,  $B \times A = \{2, 3\} \times \{1, 2, 3\} = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ 

$$(A \times B) \cap (B \times A)$$

$$= \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

Q4.

Since no two ordered pairs in  $g$  have the same first component. So,  $g$  is a function such that

$$g(1) = 1, g(2) = 3, g(3) = 5 \text{ and } g(4) = 7.$$

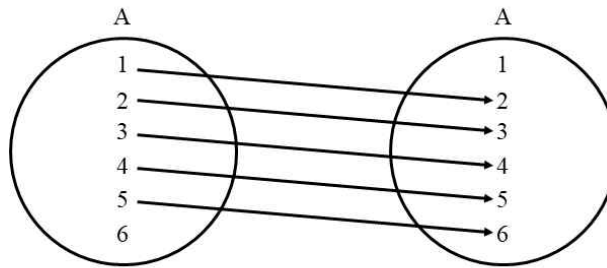
It is given that  $g(x) = \alpha(x) = \alpha x + \beta$ .

$$\therefore g(1) = 1 \text{ and } g(2) = 3 \Rightarrow \alpha + \beta = 1 \text{ and } 2\alpha + \beta = 3 \Rightarrow \alpha = 2, \beta = -1.$$

Q5.

(i) Putting  $x = 1, 2, 3, 4, 5, 6$  in  $y = x + 1$ , we get  $y = 2, 3, 4, 5, 6, 7$ .

For  $x = 6$ , we get  $y = 7$  which does not belong to set  $A$ .  
 $\therefore R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$   
 The arrow diagram representing  $R$  is as follows:



Figure

(ii) Clearly, Domain  $(R) = \{1, 2, 3, 4, 5\}$ , Range  $(R) = \{2, 3, 4, 5, 6\}$ .

Q6.

We have,

$$f(x) = \frac{3}{2 - x^2}$$

For  $f(x)$  to be real, we must have

$$2 - x^2 \neq 0 \Rightarrow x \neq \pm \sqrt{2}$$

$$\therefore \text{Domain } (f) = R - \{-\sqrt{2}, \sqrt{2}\}$$

Let  $y(x) = y$ . Then,

$$y = f(x)$$

$$\Rightarrow y = \frac{3}{2 - x^2}$$

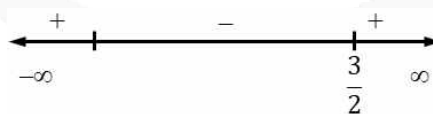
$$\Rightarrow 2y - x^2y = 3$$

$$\Rightarrow x^2y = 2y - 3$$

$$\Rightarrow x = \pm \frac{\sqrt{2y - 3}}{y}$$

Now,  $x$  will take real values other than  $-\sqrt{2}$  and  $\sqrt{2}$ , if

$$\frac{2y - 3}{y} \geq 0$$



Figure

$$\Rightarrow y \in (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$$

Hence, range  $(f) = (-\infty, 0) \cup [3/2, \infty)$ .

Q7.

Let  $b$  be an arbitrary element of  $B$ . Then,

$$(a, b) \in A \times B \text{ for all } a \in A$$

$$\Rightarrow (a, b) \in A \times C \text{ for all } a \in A$$

$$\Rightarrow b \in C$$

Thus,  $b \in B \Rightarrow b \in C$

$$\therefore B \subset C$$

$$[\because A \times B = A \times C]$$

...(i)

Now, let  $c$  be an arbitrary element of  $C$ . Then,

$$(a, c) \in A \times C \text{ for all } a \in A$$

$$\Rightarrow (a, c) \in A \times B \text{ for all } a \in A$$

$$\Rightarrow c \in B$$

$$\text{Thus, } c \in C \Rightarrow c \in B$$

$$\therefore C \subset B$$

From (i) and (ii), we get

$$B = C.$$

$$[\because A \times B = A \times C]$$

...(ii)

Q8.

We have,

$$f(x) = \sqrt{x+2} \text{ and } g(x) = \sqrt{4-x^2}$$

Clearly,  $f(x)$  is defined for

$$x+2 \geq 0 \Rightarrow x \geq -2 \Rightarrow x \in [-2, \infty)$$

$$\therefore \text{Domain } (f) = [-2, \infty)$$

$g(x)$  is defined for

$$4-x^2 \geq 0 \Rightarrow x^2-4 \leq 0 \Rightarrow (x-2)(x+2) \leq 0 \Rightarrow x \in [-2, 2]$$

$$\therefore \text{Domain } (g) = [-2, 2]$$

Now,

$$\text{Domain } (f) \cap \text{Domain } (g) = [-2, \infty) \cap [-2, 2] = [-2, 2]$$

(i)  $f+g : [-2, 2] \rightarrow R$  is given by

$$(f+g)(x) = f(x) + g(x) = \sqrt{x+2} + \sqrt{4-x^2}$$

(ii)  $f-g : [-2, 2] \rightarrow R$  is given by

$$(f-g)(x) = f(x) - g(x) = \sqrt{x+2} - \sqrt{4-x^2}$$

(iii)  $fg : [-2, 2] \rightarrow R$  is given by

$$(fg)(x) = f(x)g(x)$$

$$\Rightarrow (fg)(x) = \sqrt{x+2} \cdot \sqrt{4-x^2}$$

$$\Rightarrow (fg)(x) = \sqrt{(x+2)^2(2-x)} = (x+2)\sqrt{2-x}$$

(iv) We have,

$$g(x) = \sqrt{4-x^2}$$

$$\therefore g(x) = 0 \Rightarrow 4-x^2 = 0 \Rightarrow x = \pm 2.$$

$$\text{So, } \text{Domain} \left( \frac{f}{g} \right) = [-2, 2] - [-2, 2] = (-2, 2)$$

$$\therefore \frac{f}{g} : (-2, 2) \rightarrow R \text{ is given by}$$

$$\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+2}}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2-x}}$$

(v) We have,

$$(ff)(x) = f(x)f(x) = [f(x)]^2 = (\sqrt{x+2})^2 = x+2 \text{ for all } x \in [-2, \infty)$$

(vi) We have,

$$(gg)(x) = g(x)g(x) = [g(x)]^2 = (\sqrt{4-x^2})^2 = 4-x^2 \text{ for all } x \in [-2, 2]$$

Q9.

Here,  $f(x) = 1 - x$ ,  $x < 0$ , this gives

$$f(-4) = 1 - (-4) = 5;$$

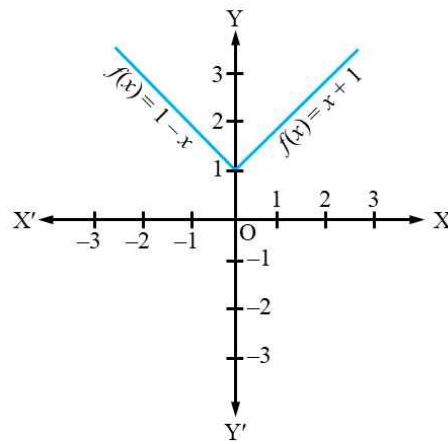
$$f(-3) = 1 - (-3) = 4,$$

$$f(-2) = 1 - (-2) = 3$$

$$f(-1) = 1 - (-1) = 2; \text{ etc,}$$

and  $f(1) = 2$ ,  $f(2) = 3$ ,  $f(3) = 4$ 

$$f(4) = 5 \text{ and so on for } f(x) = x + 1, x > 0.$$

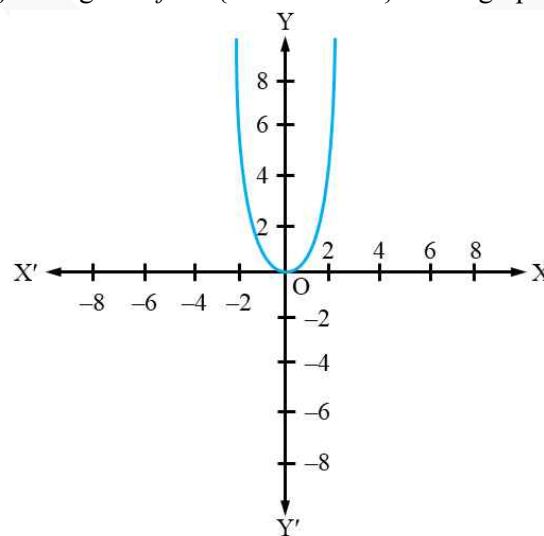
Thus, the graph of  $f$  is as shown in Figure.

Figure

Q10.

The completed Table is given below:

$x$	-4	-3	-2	-1	0	1	2	3	4
$y = f(x) = x^2$	16	9	4	1	0	1	4	9	16

Domain of  $f = \{x : x \in \mathbf{R}\}$ . Range of  $f = \{x^2 : x \in \mathbf{R}\}$ . The graph of  $f$  is given by Figure.

Figure