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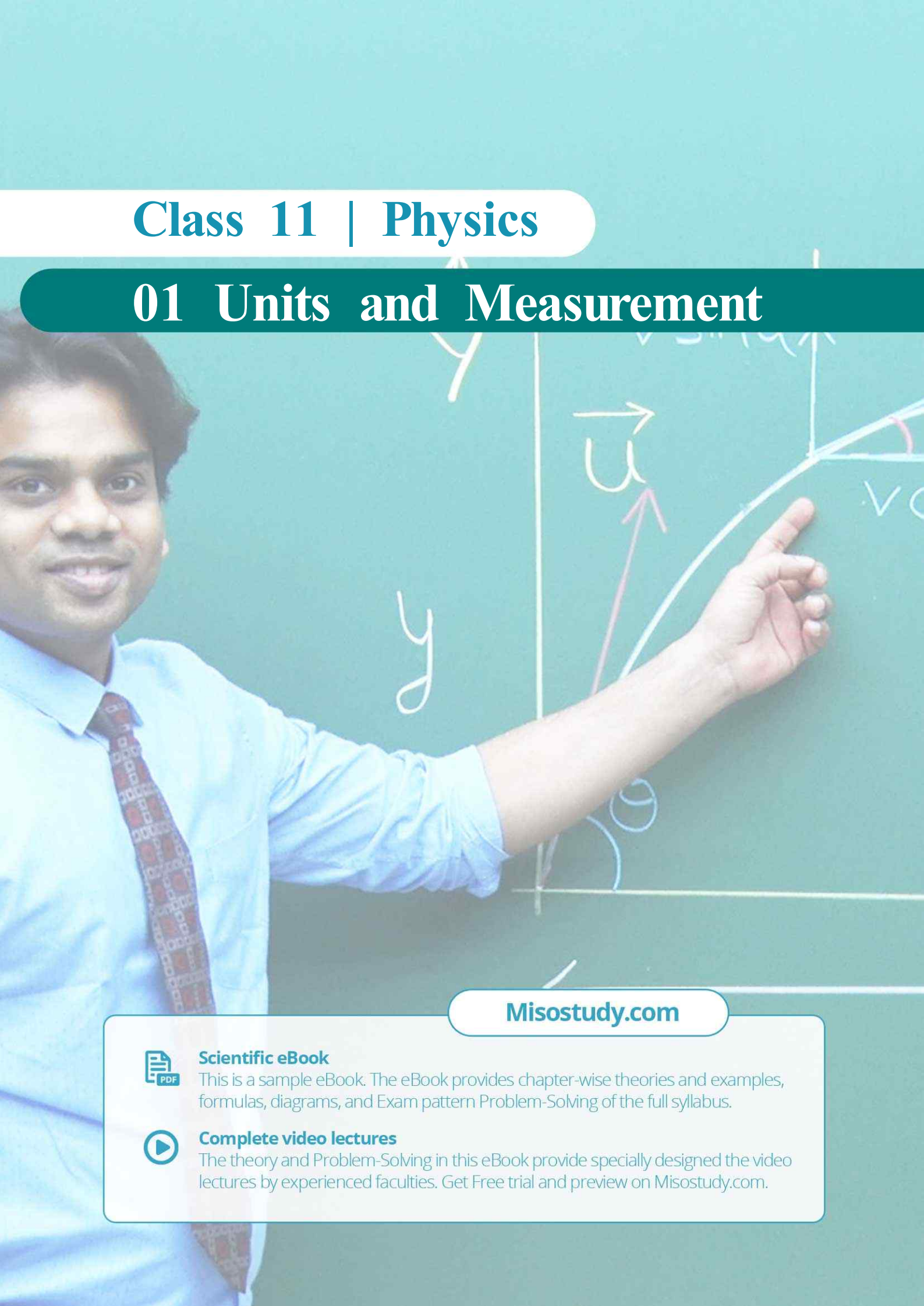
Class 11 | Physics

01 Units and Measurement



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01. Physical Quantities

All the quantities which are used to describe the laws of physics are known as *physical quantities*.

Classification : Physical quantities can be classified on the following bases :

(A) Based on their directional properties

I. Scalars : The physical quantities which have only magnitude but no direction are called *scalar quantities*.

e.g. mass, density, volume, time, etc.

II. Vectors : The physical quantities which both magnitude and direction and obey laws of vector algebra are called *vector quantities*.

e.g. displacement, force, velocity, etc.

(B) Based on their dependency

I. Fundamental or base quantities : The quantities which do not depend upon other quantities for their complete definition are known as *fundamental or base quantities*.

e.g. length, mass, time, etc.

II. Derived quantities : The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities*.

e.g. Speed (=distance/time), volume, acceleration, force, pressure, etc.

Example Classify the quantities displacement, mass, force, time, speed, velocity, acceleration, pressure and work under the following categories:

(a) base and scalar
(b) base and vector
(c) derived and scalar
(d) derived and vector

Solution

(a) mass, time
(b) displacement
(c) speed, pressure, work
(d) force, velocity, acceleration

02. Units of Physical Quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

System of Units

(i) FPS or British Engineering system : In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

(ii) CGS or Gaussian system : In this system the fundamental quantities are length, mass and time and their respective units are centimeter (cm), gram (g) and second (s).

(iii) MKS system : In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

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(iv) **International system (SI) of units :** This system is modification over the MKS system and so it is also known as *Rationalised MKS* system. Besides the three base units of MKS system four fundamental and tow supplementary units are also included in this system.

SI BASE QUANTITIES AND THEIR UNITS			
S. No.	**Physical quantity**	**Unit**	**Symbol**
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

03. Classification of Units

The units of physical quantities can be classified as follows :

(i) **Fundamental or base units**

The units of fundamental quantities are called *base units*. In SI there are seven base units.

(ii) **Derived units**

The units of derived quantities or the units that can be expressed in terms of the base units are called *derived units*.

e.g. unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{m/s}$

Some derived units are named in honour of great scientists.

e.g. unit of force – newton (N), unit of frequency – hertz (Hz), etc.

(iii) **Supplementary units**

In International System (SI) of units two *supplementary units* are also defined viz. radian (rad) for plane angle and steradian (sr) for solid angle.

- **radian :** 1 radian is the angle subtended at the centre of a circle by and arc equal in length to the radius of the circle.
- **steradian :** 1 steradian is the solid angle subtended at the centre of a sphere, by the surface of the sphere which is equal in area to the square of the radius of the sphere.

(iv) **Practical units**

Due to the fixed sizes of SI units, some *practical units* are also defined for both fundamental and derived quantities. e.g. light year (ly) is a practical unit of distance (a fundamental quantity) and horse power (hp) is a practical unit of power (a derived quantity).

Practical units may or may not belong to a particular system of units but can be expressed in any system of units.

e.g. 1 mile = 1.6 km = 1.6×10^3 m = 1.6×10^5 cm.

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Conversion factors

To convert a physical quantity from one set of units to the other, the required multiplication factor is called *conversion factor*.

Magnitude of a physical quantity = numeric value (n) × unit (u)

While conversion from one set of units to the other the magnitude of the quantity must remain same. Therefore

$n_1u_1 = n_2u_2$ or nu = constant or $n \propto \frac{1}{u}$

This is the numeric value of a physical quantity is inversely proportional to the base unit.

e.g. 1m = 100 cm = 3.28 ft = 39.4 inch
(SI) (CGS) (FPS)

Example The acceleration due to gravity is 9.8 m s^{-2}. Given its value in ft s^{-2}

Solution As 1m = 3.2 ft

$\therefore$ 9.8 m/s^2 = 9.8 × 3.28 ft/s^2 = 32.14 ft/s^2 ≈ 32 ft/s^2

04. Dimensions

Dimensions of a physical quantity are the powers for exponents to which the base quantities are raised to represent that quantity.

Dimensional formula

The dimensional formula of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

It is written by enclosing the symbols for base quantities with appropriate powers in square brackets i.e. [ ]

e. g. Dimensional formula of mass in $[M^1L^0\ T^0]$ is the dimensional formula of the force and the dimensions of force are 1 in mass, 1 in length and −2 in time

05. Applications of Dimensional Analysis

(i) To convert a physical quantity from one system of units to the other :

This is based on a fact that magnitude of a physical quantity remains same whatever system is used for measurement i.e. magnitude = numeric value (n) × unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$

Then $$n_2 = n_1\left(\frac{u_1}{u_2}\right) = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

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Here n_2 = numerical value in II system
n_1 = numerical value in I system
M_1 = unit of mass in I system
M_2 = unit of mass in II system
L_1 = unit of length in I system
L_2 = unit of length in II system
T_1 = unit of time in I system
T_2 = unit of time in II system

Example Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Solution The dimensional equation of force is $[F] = [M^1\ L^1\ T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2, corresponds to SI & CGS units respectively, then

$$n_2 = n\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1\times 1000\times 100\times 1 = 10^5 \quad \therefore$$

1 newton = 10^5 dyne.

(ii) To check the dimensional correctness of a given physical relation

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the principle of homogeneity of dimensions.

Example Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using

Solution The dimensions of LHS = the dimension of T = $[M^0\ L^0\ T^1]$

The dimensions of RHS $= \left(\frac{\text{dimensions of } \leq \text{ngth}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because$ 2π is a dimensionless constant)

$$= \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T^2]^{1/2} = [T] = [M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) To derive relationship between different physical quantities

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Example It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

We have $[T] = [G]^a[M]^b[R]^c$

Solution $[M]^0[L]^0[T]^1 = [M]^{-a}[L]^{3a}[T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a}[L]^{c+3a}[T]^{-2a}$

Comparing the exponents

For $[T]: 1 = -2a \Rightarrow a = -\frac{1}{2}$ For $[M]: 0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For $[L]: 0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

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Putting the values we get $T \propto G^{-1/2} M^{-1/2} R^{3/2} \Rightarrow T \propto \sqrt{\frac{R^3}{GM}}$

The actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Dimensions of trigonometric, exponential, logarithmic function etc.

All trigonometric, exponential and logarithmic functions and their arguments are dimensionless.

NOTE ☞ Trigonometric function sinθ and its argument θ are dimensionless.

06. Limitations of this Method

- In Mechanics the formula for a physical quantity depending one more than three physical quantities cannot be derived. It can only be checked.
- This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions cant't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.
- The relation derived from this method gives no information about the dimensionless constants.
- If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
- It gives no information whether a physical quantity is a scalar or a vector.

07. Significant Figures or Digits

The significant figure (SF) in a measurement are the figure or digits that are known with certainity plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figure obtained in a measurement, greater is its accuracy and vice versa.

Rules to find out the number of significant figures

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g. 00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

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VI Rule : The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. e.g. m = 100 kg has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = 0.123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

Rules for arithmetical operations with significant figures

I Rule : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. $12.587 - 12.5 = 0.087 = 0.1$ ($\because$ second term contain lesser i.e. one decimal place)

II Rule : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $4.0 \times 0.12 = 0.484 = 0.48$

- To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in scientific notation (in the power of 10). In this notation every number is expressed in the form a × 10b, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).
- The change in the unit of measurement of a quantity does not effect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

Example Write down the number of significant figures in the following.

(a) 165
(b) 2.05
(c) 34.000 m
(d) 0.005
(e) 0.02340 N m^{-1}
(f) 26900
(g) 26900 kg

Solution

(a) 165	3 SF (following rule I)
(b) 2.05	3 SF (following rule I & II)
(c) 34.000 m	5 SF (following rule I & V)
(d) 0.005	1 SF (following rules I & IV)
(e) 0.02340 N m^{-1}	4 SF (following rule I, IV & V)
(f) 26900	3 SF (see rule VI)
(g) 26900 kg	5 SF (see rule VI)

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08. Rounding Off

To represent the result of any computation containing more than one uncertain digit, it is rounded off to appropriate number of significant figures.

Rules for rounding off the numbers :

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II Rule : If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

III Rule : If the digit to be rounded off is 5 than the preceding digit is increased by one if it odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

Example The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Solution length (l) = 4.234 m breadth (b) = 1.005 m

thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore area of the sheet = $2\ (l \times b + b \times t + t \times l)$

$= 2\ (4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2$

$= 2\ (4.3604739)\ m^2 = 8.720978\ m^2$

Since area can contain a max^m of 3 SF (Rule II of article 4.2) therefore, rounding off, we get

Area = $8.72\ m^2$

Like wise volume = $l \times b \times t = 4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore, rounding off, we get

Volume = $0.0855\ m^3$

09. Order of Magnitude

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and, is increased by one if it is 5 or more than 5.

- When a number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 i.e. $0.5 \le N/10^x < 5$

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Example Order of magnitude of the following values can be determined as follows :

(a) 49 $= 4.9 \times 10^1 \approx 10^1$

Solution $\therefore$ Order of magnitude = 1

(b) 51 $= 5.1 \times 10^1 \approx 10^2$

Solution $\therefore$ Order of magnitude = 2

(c) 0.049 $= 4.9 \times 10^{-2} \approx 10^{-2}$

Solution $\therefore$ Order of magnitude = −2

(d) 0.050 = $5.0 \times 10^{-2} \approx 10^{-1}$

Solution $\therefore$ Order of magnitude = −1

(e) 0.051 = $5.1 \times 10^{-2} \approx 10^{-1}$

Solution $\therefore$ Order of magnitude = −1

- **Accuracy, Precision of Instruments and Errors in Measurement**

Accuracy and Precision

The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called error. Every calculated quantity which is based on measured value, also has an error. Every measurement is limited by the reliability of the measuring instrument and skill of the person making the measurement. If we repeat a particular measurement, we usually do not get precisely the same result as each result is subjected to some experimental error. This imperfection in measurement can be described in terms of accuracy and precision. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured, we can illustrate the difference between accuracy and precision with help of a example. Suppose the true value of a certain length is 1.234 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 1.1cm, while in another experiment using a measuring device of greater resolution of 0.01m, the length is determined to be 1.53cm. The first measurement has move accuracy (as it is closer to the true value) but less precision (as resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

10. Errors

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Errors may arise from different sources and are usually classified as follows

Systematic or Controllable Errors

Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to the known causes these errors can be minimised. Systematic errors can further be classified into three categories

(i) **Instrumental errors :-** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(ii) **Environmental errors :-** These are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust vibrations or magnetic and electrostatic fields.

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(iii) Observational errors :- These errors arise due to improper setting of the apparatus or carelessness in taking observations.

Random Errors

These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random errors can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the observations. This mean value would be very close to the most accurate reading.

NOTE ☞ If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times.

Gross Errors : Gross errors arise due to human carelessness and mistakes in reading the instruments or calculating and recording the measurement results.

For example :-

(i) Reading instrument without proper initial settings.

(ii) Taking the observations wrongly without taking necessary precautions.

(iii) Exhibiting mistakes in recording the observations.

(iv) Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

11. Representation of Errors

Errors can be expressed in the following ways

Absolute Error (Δa) : The difference between the true value and the individual measured value of the quantity is called the absolute error of the measurement.

Suppose a physical quantity is measured n times and the measured values are a_1, a_2, a_3 .....a_n. The arithmetic mean (a_m) of these values is

$$a_m = \frac{a_1 + a_2 + a_3 + ...a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i \qquad ...(i)$$

If the true value of the quantity is not given then mean value (a_m) can be taken as the true value. Then the absolute errors in the individual measured values are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
.....
.....
$$\Delta a_n = a_m - a_n$$

The arithmetic mean of all the absolute errors is defined as the final or mean absolute error $(\Delta a)_m$ or Δa of the value of the physical quantity a

$$(\Delta a_m) = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i| \qquad ...(ii)$$

So if the measured value of a quantity be 'a' and the error in measurement be Δa, then the true value (a_t) can be written as

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$$a_t = a \pm \Delta a \quad \text{...(iii)}$$

Relative or Fractional Error : It is defined as the ratio of the mean absolute error ($(\Delta a)_m$ or $\overline{\Delta a}$) to the true value or the mean value (a_m or $\bar{a}$) of the quantity measured.

$$\text{Relative or fractional error} = \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{(\Delta a)_m}{a_m} \text{ or } \frac{\overline{\Delta a}}{\bar{a}} \quad \text{...(iv)}$$

When the relative error is expressed in percentage, it is known as percentage error, percentage error = relative error × 100

$$\text{or} \quad \text{percentage error} = \frac{\text{mean absolute error}}{\text{true value}} \times 100\% = \frac{\overline{\Delta a}}{a} \times 100\% \quad \text{...(v)}$$

12. Propagation of Errors in Mathematical Operations

Rule I : The maximum absolute error in the sum or difference of the two quantities is equal to the sum of the absolute errors in the individual quantities.

If $X = A + B$ or $X = A - B$ and if ± ΔA and ± ΔB represent the absolute errors in A and B respectively, then the maximum absolute error in $X = \Delta X = \Delta A + \Delta B$ and

$$\text{Maximum percentage error} = \frac{\Delta X}{X} \times 100 \quad \text{...(i)}$$

The result will be written as $X \pm \Delta X$ (in terms of absolute error)

or $X \pm \frac{\Delta X}{X} \times 100\%$ (in terms of percentage error)

Rule II : The maximum fractional or relative error in the product or division of quantities is equal to the sum of the fractional or relative errors in the individual quantities.

If $X = A \times B$ or $X = A/B$

$$\text{then} \quad \frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \quad \text{...(ii)}$$

Rule III : The maximum fractional error in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

$$\text{If} \quad X = A^n \quad \text{then} \quad \frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right) \quad \text{...(viii)}$$

$$\text{If} \quad X = A^p B^q C^r \quad \text{then} \quad \frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$$

$$\text{If} \quad X = \frac{A^p B^q}{C^r} \quad \text{then} \quad \frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$$

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IMPORTANT POINTS

- Systematic errors are repeated consistently with the repetition of the experiment and are produced due to improper conditions or procedures that are consistent in action whereas random errors are accidental and their magnitude and sign cannot be predicated from the knowledge of the measuring system and conditions of measurement.
  Systematic errors can therefore be minimised by improving experimental techniques, selecting better instruments and improving personal skills whereas random errors can be minimised by repeating the observation several times.
- Mean absolute error has the units and dimensions of the quantity itself whereas fractional or relative error is unitless and dimensionless.
- Absolute errors may be positive in certain cases and negative in other cases.

Example The initial and final temperatures of water as recorded by an observer are (40.6 ± 0.2)ºC and (78.3 ± 0.3)ºC. Calculate the rise in temperature with proper error limits.

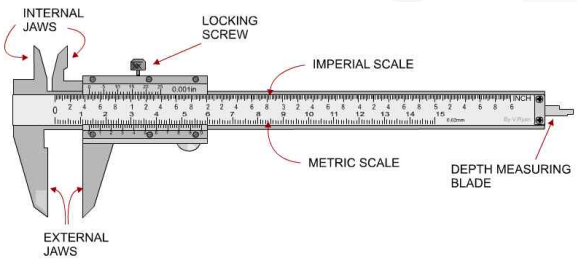
Solution Given θ_1 = (40.6 ± 0.2)ºC and θ_2 = (78.3 ± 0.3)ºC
Rise in temp. $\theta = \theta_2 - \theta_1$ = 78.3 − 40.6 = 37.7ºC.
$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2)$ = ± (0.2 + 0.3) = ± 0.5ºC ∴ rise in temperature = (37.7 ± 0.5)ºC

13. Least Count

The smallest value of a physical quantity which can be measured accurately with and instrument is called the least count (L. C.) of the measuring instrument.

Least Count of Vernier Callipers

Suppose the size of one main scale division (M.S.D.) is M units and that of one vernier scale division (V. S. D.) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' vernier scale divisions.



$$aM = bV \Rightarrow V = \frac{a}{b}M$$

$$\therefore \quad M - V = M - \frac{a}{b}M \text{ or } \quad M - V = \left(\frac{b-a}{b}\right)M$$

The quantity (M− V) is called vernier constant (V. C.) or least count (L. C.) of the vernier callipers.

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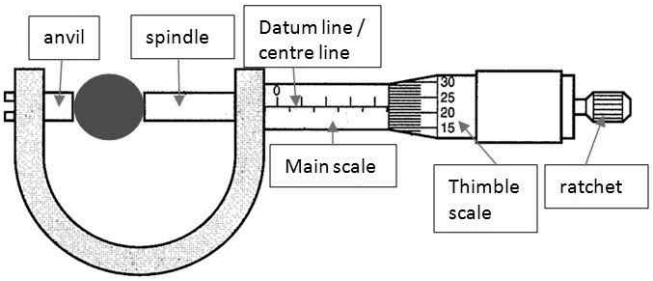
$$\boxed{\text{L.C.} = \text{M} - \text{V} = \left(\frac{b-a}{b}\right)\text{M}}$$

Least Count of screw gauge or spherometer



$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total number of divisions on the circul scale}}$$

where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e.

$$\text{Pitch} = \frac{\text{Distance moved by the screw on the linear scale}}{\text{No. of full rotations given}}$$

NOTE ☞ With the decrease in the least count of the measuring instrument, the accuracy of the measurement increases and the error in the measurement decreases.

NOTE ☞ The final absolute error in this type of questions is taken to be equal to the least count of the measuring instrument.

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JEE Main Pattern
Exercise (1)

1. Force F is given in terms of time t and distance x by $F = A\sin Ct + B\cos Dx$. Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by
   (a) $[MLT^{-2}, M^0L^0T^{-1}]$
   (b) $[MLT^{-2}, M^0L^{-1}T^0]$
   (c) $[M^0L^0T^0, M^0LT^{-1}]$
   (d) $[M^0LT^{-1}, M^0L^0T^0]$

2. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
   (a) $v^2 = k\lambda^{-1}g^{-1}\rho^{-1}$
   (b) $v^2 = kg\lambda$
   (c) $v^2 = kg\lambda\rho$
   (d) $v^2 = k\lambda^3 g^{-1}\rho^{-1}$
   Where k is a dimensionless constant

3. In a vernier callipers, N divisions of the main scale coincide with N + m divisions of the vernier scale. What is the value of m for which the instrument has minimum least count?
   (a) 1
   (b) N
   (c) $\frac{N}{10}$
   (d) $N/2$

4. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is
   (a) $(0.34 \pm 0.02)cm$
   (b) $(0.17 \pm 0.02)cm$
   (c) $(0.17 \pm 0.01)cm$
   (d) $(0.34 \pm 0.01)cm$

5. The vernier constant of a travelling microscope is 0.001 cm. If 49 main scale divisions coincide with 50 vernier scale divisions, then the value of 1 main scale division is
   (a) 0.1 mm
   (b) 0.5 mm
   (c) 0.4 mm
   (d) 1 mm

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6. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is
   (a) $[MLT^{-1}]$
   (b) $[ML^2T^{-2}]$
   (c) $[MLT^{-2}]$
   (d) $[ML^{-1}T^{-2}]$

7. In the relation
$$p=\frac{\alpha}{\beta}e^{-\frac{\alpha Z}{k\theta}}$$
   p is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
   (a) $[M^0L^2T^0]$
   (b) $[ML^2T]$
   (c) $[ML^0T^{-1}]$
   (d) $[M^0L^2T^{-1}]$

8. The period of oscillation of a simple pendulum is $T=2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is
   (a) 3%
   (b) 2%
   (c) 1%
   (d) 5%

9. The current voltage relation of diode is given by $I=(e^{1000V/T}-1)mA$, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?
   (a) 0.2 mA
   (b) 0.02 mA
   (c) 0.5 mA
   (d) 0.05 mA

10. A wire has a mass $(0.3\pm 0.003)g$, radius $(0.5\pm 0.005)mm$ and length $(6\pm 0.06)cm$. The maximum percentage error in the measurement of its density is
   (a) 1
   (b) 2
   (c) 3
   (d) 4

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(c)	(b)	(a)	(c)	(b)
Q6	Q7	Q8	Q9	Q10
(d)	(a)	(a)	(a)	(d)

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JEE Advanced Pattern
Exercise (1)

1. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length,
T = Time and A = electric current, then
(a) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$
(b) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
(c) $[\varepsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$
(d) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^2]$

2. Which of the following sets have different dimensions?
(a) Pressure, Young's modulus, Stress
(b) Emf, Potential difference, Electric potential
(c) Heat, Work done, Energy
(d) Dipole moment, Electric flux, Electric field

3. A quantity X is given by $\varepsilon_0 L\frac{\triangle V}{\triangle t}$, where ε_0 is the permittivity of free space, L is a length, $\triangle V$ is a potential difference and $\triangle t$ is a time interval. The dimensional formula for X is the same as that of
(a) resistance
(b) charge
(c) voltage
(d) current

4. A wire has mass (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is
(a) 1
(b) 2
(c) 3
(d) 4

5. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is
(a) 0.9%
(b) 2.4%
(c) 3.1%
(d) 4.2%

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(Q 6 to 7) More than One Correct

6. The SI unit of the inductance, the henry can by written as
   (a) weber/ampere
   (b) volt-second/ampere
   (c) joule/(ampere)^2
   (d) ohm-second

7. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current.
   (a) $[\varepsilon_0] = [M^{-1}L^{-3}T^2 I]$
   (b) $[\varepsilon_0] = [M^{-1}L^{-3}T^4 I^2]$
   (c) $[\mu_0] = [MLT^{-2} I^{-2}]$
   (d) $[\mu_0] = [ML^2 T^{-1} I]$

(Q 8 to 9) Match the Columns

8. Match List I with List II and select the correct answer using the codes given below the lists.

	Column I		Column II
(a)	Boltzmann constant	p.	$[ML^2T^{-1}]$
(b)	Coefficient of viscosity	q.	$[ML^{-1}T^{-1}]$
(c)	Planck constant	r.	$[MLT^{-3}K^{-1}]$
(d)	Thermal conductivity	s.	$[ML^2T^{-2}K^{-1}]$

9. Column I gives three physical quantities. Select the appropriate units for the choices given in **Column II**. Some of the physical quantities may have more than one choice.

Column I	Column II
(a) Capacitance	(i) Ohm-second
(b) Inductance	(ii) coulomb^2-joule^{-1}
(c) Magnetic induction	(iii) coulomb (volt)$^{-1}$,
	(iv) newton (ampere metre)$^{-1}$,
	(v) volt-second (ampere)$^{-1}$

(Q 10) Integer Type

10. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(b)	(d)	(d)	(d)	(c)
Q6	**Q7**	**Q8**	**Q9**	**Q10**
(a), (b), (c), (d)	(b), (c)	(a)→s, (b)→q, (c)→p, (d)→r	(a)→ (ii), (iii) (b)→ (i), (v) (c)→ (iv)	3

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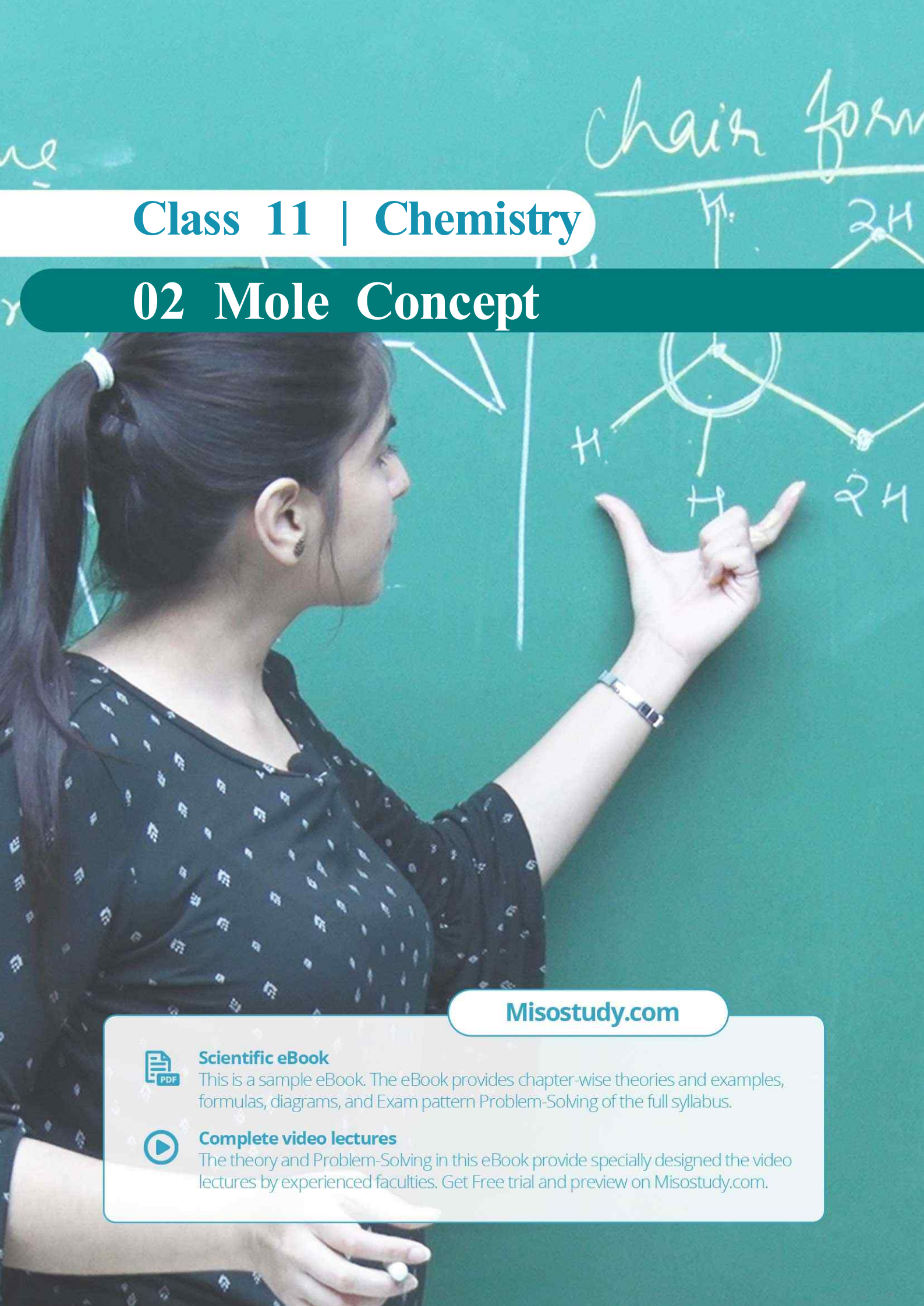

Class 11 | Chemistry

02 Mole Concept

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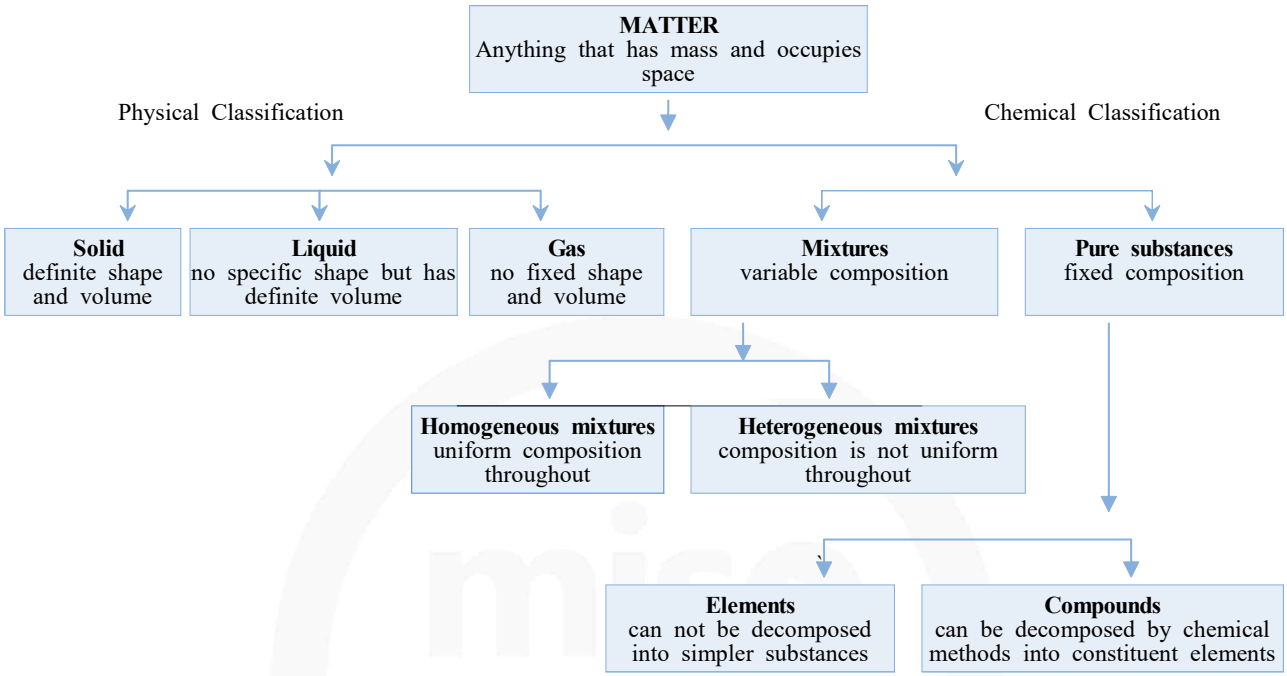
01. Classification of matter

Chemistry deals with the composition, structure and properties of matter. These aspects can be best described and understood in terms of basic constituents of matter: **atoms** and **molecules.** That is why chemistry is called the science of atoms and molecules.



Matter

The thing which occupy space and have mass, which can be felt by our five sense is called as matter. Matter is further classified into two categories :

a. Physical classification
b. Chemical classification

02. Prefixed Used With Units

The S.I. system recommends the multiples such as 10^3, 10^6, 10^9 etc. and fraction such as 10^{-3}, 10^{-6}, 10^{-9} etc. i.e. the powers are the multiples of 3. These are indicated by special prefixes. These along with some other fractions or multiples in common use, along with their prefixes are given below in Table and illustrated for length (m).

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※TABLE : SOME COMMONLY USED PREFIXES WITH THE BASE UNIT

Prefix	Symbol	Multiplication Factor	Example
deci	d	10^{-1}	1 decimetre (dm) = 10^{-1} m
centi	c	10^{-2}	1 centimetre (cm) = 10^{-2} m
milli	m	10^{-3}	1 millimetre (mm) = 10^{-3} m
micro	μ	10^{-6}	1 micrometre (μm) = 10^{-6} m
nano	n	10^{-9}	1 nanometre (nm) = 10^{-9} m
pico	p	10^{-12}	1 picometre (pm) = 10^{-12} m
femto	f	10^{-15}	1 femtometre (fm) = 10^{-15} m
atto	a	10^{-18}	1 attometre (am) = 10^{-18} m
deka	da	10^{1}	1 dekametre (dam) = 10^{1} m
hecto	h	10^{2}	1 hectometre (hm) = 10^{2} m
kilo	k	10^{3}	1 kilometre (km) = 10^{3} m
mega	M	10^{6}	1 megametre (Mm) = 10^{6} m
giga	G	10^{9}	1 gigametre (Gm) = 10^{9} m
tera	T	10^{12}	1 teremetre (Tm) = 10^{12} m
peta	P	10^{15}	1 petametre (Pm) = 10^{15} m
exa	E	10^{18}	1 exametre (Em) = 10^{18} m

As volume is very often expressed in litres, it is important to note that the equivalence in S.I. units for volume is as under: 1 litre (1 L) = 1 dm^3 = 1000 cm^3 and 1 millilitre (1 ml) = 1 cm^3 = 1 cc

Example Convert 2 atm into cm of Hg.

Solution 2 atm = 2 × 76 cm of Hg = 152 cm of Hg {1 atmosphere = 76 cm of Hg

03. Different types of masses

One mole

Avogadro's Number (N_A)=6.023×10^{23}. It is the number of atoms present in exactly 12 g of (C^{12}) isotope.

Atomic Weight (*A*)

Atomic weight is the relative weight of one atom of an element with respect to a standard weight.

$$A=\frac{\text{Weight of one atom of an element}}{\frac{1}{12}\text{th part by weight of an atom of }(C^{12})\text{ isotope}}$$

amu (atomic mass unit)

$$1\text{ amu}=\frac{1}{12}\text{th part by weight of an atom of }(C^{12})\text{ isotope}$$

$$=\frac{1}{N_A}g=1.66\times10^{-24}g$$

Atomic weight (*A*)×amu =Absolute atomic weight.

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NOTE ☞ Atomic weight is a relative weight that indicates the relative heaviness oof one atom of an element with respect to amu weight. Atomic weight has no unit because it is the ratio of weights. One mole of an amu = 1.00 g.

Change of Scale for Atomic Weight

If an amu is defined differently as (1/x)th part by weight of an atom of (C^{12})isotope rather (1/12)th part then the atomic weight (A') can be derived as:

$$A' = A\left(\frac{x}{12}\right)$$

Where, A = conventional atomic weight

Molecular Weight (MW)

Like atomic weight, it is the relative weight of a molecule or a compound with respect to amu weight.

$$\text{Molecular Weight} = \frac{\text{Weight of one molecule of a compound}}{\frac{1}{12}\text{th part by weight of an atom of } C^{12} \text{ isotope}}$$

Gram Atomic, Gram Molecular Weight (M)

It is the weight of 1.0 mole (Avogadro's numbers) of atoms, molecules or ions in gram unit. $M = A$ amu $\times$ Avogadro number = A gram Hence, gram molecular weight (M) is numerically equal to the atomic weight or (molecular weight) in gram unit because

1.0 mole of amu is 1.0 g.

Example A piece of Cu contain 6.022×10^{24} atoms. How many mole of Cu atoms does it contain?

Solution No. of mole $= \dfrac{6.022 \times 10^{24}}{N_A} = \dfrac{6.022 \times 10^{24}}{6.022 \times 10^{23}} = 10$ mole

04. Law of conservation of mass (Lavoisier-1774):

In any physical or chemical change, mass can neither be created nor be destroyed.

It means:

Total mass of the reactants = total mass of the products.

This relationship holds good when reactants are completely converted into products.

In case the reacting material are not completely consumed the relationship will be-

Total mass of the reactants = Total mass of the products + mass of unreacted reactants.

Example 1.7 gram of silver nitrate dissolved in 100 gram of water is taken. 0.585 gram of sodium chloride dissolved in 100 gram of water is added it and chemical reaction occurs. 1.435 gm of AgCl and 0.85 gm $NaNO_3$ are formed. Show that these results illustrate the law of conservation of mass.

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Solution Total masses before chemical change
= mass of $AgNO_3$ + mass of NaCl + mass of water
= 1.70 + 0.585 + 200g
= 202.285 g
Total masses after the chemical reaction
= mass of AgCl + mass of $AgNO_3$ + mass of water
= 1.435 + 0.85 + 200
= 202.258 g
Then, in this chemical change
Total masses of reactants = Total masses of product

05. Law of constant composition : [proust 1799]

A chemical compound always contains the same element combined together in fixed proportion by mass.

Example 1.08 gram of Cu wire was allowed to react with nitric acid. The resulting solution was dried and ignited when 1.35 gram of copper oxide was obtained. In another experiment 1.15 gram of copper oxide was heated in presence of Hydrogen yielding 0.92 gram of copper. Show that the above data are in accordance with law of constant composition?

Solution % of "Cu" in copper oxide in 1st case $= \frac{1.08}{1.35} \times 100$
$= 80\%$
% of oxygen = 20%
% of "Cu" in copper oxide in 2nd case $= \frac{0.92}{1.15} \times 100$
$= 80\%$
% of oxygen = 20%

06. Law of multiple proportion : [Dalton 1806]

When two elements combine to form two or more compounds, the different masses of one element which combine with a fixed mass of the other element, bear a simple ratio to one another.

Example Two compounds each containing only tin and oxygen had the following composition.

	Mass % of Tin	**Mass % of oxygen**
Compound A	78.77	21.23
Compound B	88.12	11.88

Show that these data illustrate the law of multiple proportion?

Solution **In compound A**
21.23 parts of oxygen combine with 78.77 parts of tin.
1 part of oxygen combine with $\frac{78.77}{21.23} = 3.7$ parts of Sn.

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In compound B

11.88 parts of oxygen combine with 88.12 parts of tin.

1 part of oxygen combine with $\frac{88.12}{11.88} = 7.4$ parts of tin.

Thus the mass of Tin in compound A and B which combine with a fixed mass of oxygen are in the ratio 3.7:7.4 or 1:2. This is a simple ratio. Hence the data illustrate the law of multiple proportion.

07. Law of reciprocal proportion : [Richter 1794]

When two different elements combine with the same mass of a third element, the ratio on which the do so will be same or simple multiple if both directly combined with each other.

Example The % composition of NH_3, H_2O and N_2O_3 is as given below:

$NH_3 \rightarrow$ 82.35% N and 17.65 H.

$H_2O \rightarrow$ 88.9% O and 11.1 H

$N_2O_3 \rightarrow$ 63.15% O and 36.85% N

On the basis of above data prove the law of reciprocal proportion?

Solution 63.15 parts of oxygen combine with 36.85 parts of "N"

Therefore, 88.9 part of oxygen combine with $\left(\frac{36.85}{63.15} \times 88.9\right) = 51.87$ part of "Nitrogen"

Therefore ratio is $\frac{11.1}{11.1} : \frac{51.87}{11.1} = 1 : 4.67$

Now compare with the ratio of Nitrogen and Hydrogen in NH_3

$$\frac{17.65}{17.65} : \frac{82.35}{17.65} = 1 : 4.67$$

Hence the Law of reciprocal proportion is verified

08. Gay- Lussac's law of gaseous volumes [Gay-Lussac-1808]

When gases combined or produced in a chemical reaction, they do so in a simple ratio by volume provided all the gases are at same temperature and pressure.

09. Limiting Reagent

It is the reagent that is consumed completely during a chemical reaction. If the supplied mass ratio of reactants are not stoichiometric ratio, one of the reagent is consumed completely leaving parts of others unreacted. One that is consumed completely is known as limiting reagent.

'Limiting reagent determine the amount of product in a given chemical reaction'

Example If 20 gm of $CaCO_3$ is treated with 20 gm of HCl, how many grams of CO_2 can be generated according to following reaction?

$CaCo_3(g) + 2HCl(aq) \rightarrow CaCl_2(aq) + H_2O(\ell) + CO_2(g)$

Solution $CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

1 mole $CaCO_3 \rightarrow$ 2 mole HCl

$\therefore$ 100 g $CaCO_3$ combine $\rightarrow 2 \times 36.5$ g HCl

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$\therefore$ 20 g $\rightarrow \frac{2\times36.5\times20}{100} = 14.6\,HCl$

$CaCO_3$ completely consumes in the reaction and HCl is in excess.
Therefore,
$CaCO_3 \rightarrow$ Limiting reagent
HCl $\rightarrow$ Excess reagent
Now
100 g $CaCO_3$ given 1 mole CO_2 (44g CO_2)
20 g $CaCO_3$ will give $\frac{20\times40}{100}$ = 8 g CO_2

10. Percentage yield

In general, when a reaction is carried out on the laboratory we do not obtain the theoretical amount of product. The amount of product that is actually obtained is called the actual yield. Knowing the actual yield and theoretical yield, the % yield can be calculated by the following formula-

$$\text{Percentage yield} = \frac{\text{Actual yield}}{\text{Theoritical yield}}\times100\%$$

Example For the reaction
$CaO+2HCl \rightarrow CaCl_2+H_2O$
1.12 gram of CaO is reacted with excess of hydrochloric acid and 1.85 gm $CaCl_2$ is formed. What is the % yield of the reaction?

Solution 1 mole CaO gives 1 mole $CaCl_2$
56 g CaO gives 111 g $CaCl_2$
1.12 g CaO will give $\frac{111\times1.12}{56}$ g $CaCl_2$ = 2.22 g $CaCl_2$
Now
% yield = $\frac{\text{Actual yield}}{\text{Theoretical yield}}\times100$
Actual yield = 1.85 gm
Theoretical yield = 2.22 gm
% yield = $\frac{1.85}{2.22}\times100$ = 83.33%

11. Percentage Purity

Depending upon the mass of the product, the equivalent amount of reactant present can be determined with the help of given chemical equation. Knowing the actual amount of the reactant taken and the amount calculated with the help of a chemical equation, the purity van be determined, as

$$\text{Percentage purity} = \left[\frac{\text{Amount of reactant calculated from the chemical equation}}{\text{Actual amount of reactant taken}}\right]\times100\%$$

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Example Calculate the amount of (CaO) in kg that can be produced by heating 200 kg lime stone that is 90% pure $CaCO_3$.

Solution $CaCO_3 \rightarrow CaO + CO_2$

1 mole $CaCO_3$ gives 1 mole CaO

100 g $CaCO_3$ gives 56 g CaO

$$\left(\frac{200\times90\times1000}{100}\right)\text{g } CaCO_3 \text{ gives} = \frac{180000\times56}{100}\text{g CaO}$$

$$= \frac{1800\times56}{100}\text{g CaO}$$

$$= \frac{1008}{10}\text{g CaO}$$

$$= 100.8 \text{ g CaO}$$

12. Types of Average masses

Average Atomic Mass

Average atomic mass =

Let a sample contains n_1 mole of atomic mass M_1 and n_2 mole of atoms with atomic mass M_2 then

$$M_{av} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

Average Molecular Mass

Average molecular mass =

Let a sample contains n_1 mole of molecules with molecular mass M_1 and n_2 mole of molecules with molecular mass M_2 ,then

$$M = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

Example Find the average atomic mass of a mixture containing 25% by mole Cl^{37} and 75% by mole Cl^{35}?

Solution $n_1 = 25$ $n_2 = 75$ $M_1 = 37$ $M_2 = 35$

$$M_{av} = \frac{25\times37+75\times35}{25+75} = 35.5$$

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13. Empirical & molecular formula

The empirical formula of a compound is a chemical formula showing the relative number of atoms in the simplest ratio. An empirical formula represents the simplest. whole number ratio of various atoms present in a compound.

The molecular formula gives the actual number of atoms of each element in a molecule. The molecular formula shows the exact number of different types of atoms present in a molecule of a compound. The molecular formula is an integral multiple of the empirical formula.

i.e. molecular formula=empirical formula $\times$ n where $n = \frac{\text{molecular formula mass}}{\text{empirical formula mass}}$

Example An organic substance containing carbon, hydrogen and oxygen gave the following percentage composition.

C=40,684% ; H=5,085% ; and O=54,228%

The molecular weight of the compound is 118 g. Calculate the molecular formula of the compound.

Solution **Step-1 :** To calculate the empirical formula of the compound.

Element	Symbol	percentage of element	At. mass of element	Relative no. of atoms= $\frac{\text{Percentage}}{\text{At. mass}}$	Simplest atomic ratio	Simplest whole no. atomic ratio
Carbon	C	40.678	12	$\frac{40.678}{12}=3.390$	$\frac{3.390}{3.389}=1$	2
Hydrogen	H	5.085	1	$\frac{5.085}{1}=5.085$	$\frac{5.085}{3.389}=1.5$	3
Oxygen	O	54.228	16	$\frac{54.228}{16}=3.389$	$\frac{3.389}{3.389}=1$	2

$\therefore$ Empirical Formula is $C_2\ H_3\ O_2$

Step-2 : To calculate the empirical formula mass. The empirical formula of the compound is $C_2\ H_3\ O_2$.

Empirical formula mass = $(2\times12)+(3\times1)+(2\times16)=59$.

Step-3 : To calculate the value of 'n' $n=\frac{\text{Molecular mass}}{\text{Empirical formula mass}}=\frac{118}{59}=2$

Step-4 : To calculate the molecular formula of the salt.

Molecular formula = $n=(\text{Empirical formula})=2\times C_2H_3O_2=C_4H_6O_4$

Thus the molecular formula is $C_4H_6O_4$

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14. Vapour Density

Some times in numericals molecular mass of volatile substance is not given, instead vapour density is given. Vapour density van be defined as

$$\text{V.D.} = \frac{\text{Density of gas at a given T and P}}{\text{Density of } H_2 \text{ at same T and P}}$$

or, $\text{V.D.} = \frac{M_{gas}}{2}$

$M_{gas} = 2 \times \text{V.D.}$

15. Eudiometry – Gas Analysis

The study of gaseous reactions is done in a eudiometer tube with the help of Gay-Lussac's law and Avogadro's law. Eudiometer tube is a closed graduated tube open at one end. The other end is a closed one which is provided with platinum terminals for passing electricity for electric spark, through the known volume of mixture of gases and known volume of oxygen gas. Volume of CO_2 formed is determined by absorbing in KOH solution, O_2 is determined by dissolving unreacted O_2 in alkaline pyrogallol and water vapours formed are determined by nothing contraction in volume caused due to cooling.

16. Avogadro's Law

In 1812, Amadeo Avogadro stated that samples of different gases which contain the same number of molecules (any complexity, size, shape) occupy the same volume at the same temperature and pressure.
For ideal gas at constant Temperature & Pressure, pressure is directely proportional to no. of moles

17. Some Absorbents of Gases

The absorbent which is used for specific gas is listed below

Absorbent	Gas or gases absorbed
Turpentine oil	O_3
Alkaline pyrogallol	O_2
Ferrous sulphate solution	NO
Heated magnesium	N_2
Heated palladium	H_2
Ammonical coprous chloride	O_2, CO, C_2H_2 or $CH \equiv CH$
Copper sulphate solution	H_2S, PH_3, AsH_3
Conc. H_2SO_4	H_2O I.e., moisture, NH_3.
NaOH or KOH solution	CO_2, NO_2, SO_2, X_2, all acidic oxides

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18. Volume Expansion and Contraction In The Eudiometer Tube

$$aA(g) + bB(g) \qquad cC(g) + dD(g)$$

$\triangle n_g$ = No. of gaseous products–No. of gaseous reactants = (c+d)–(a+b)

(a) If $\triangle n_g > 0$, then expansion will occur

(b) If $\triangle n_g = 0$, No contraction/expansion (volume remains constant)

(c) If $\triangle n_g < 0$, then contraction will occur

19. Assumptions

(i) All gases are assumed to be ideal.

(ii) Nitrogen gas formed during reaction will not react with any other gas.

(iii) The volume of solids and liquids are negligible in comparision to the volume of gas.

20. General Reactions for Combustion of Organic Compounds

(i) When an organic compound is hydrocarbon :

$$C_xH_Y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

(ii) When an organic compound contain carbon, hydrogen and oxygen :

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

(iii) When an organic compound contain carbon, hydrogen and nitrogen :

$$C_xH_yN_z + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2$$

Example 10 ml of a mixture of CO, CH_4 and N_2, exploded with excess of oxygen, gave a contraction of 6.5 ml. There was a further contraction of 7 ml. when the residual gas was treated with KOH. What is the composition of the original mixture?

Solution $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

x ml $\quad \frac{x}{2}$ ml $\qquad x$ ml

y ml $\quad 2y$ ml $\qquad y$ ml

volume of oxygen used = $2y + \frac{x}{2}$ ml

Total volume of all gases before combustion = $10 + 2y + x/2$

21. Percentage

Concentration of solution is the amount of solute dissolved in a known amount of the solvent or solution. The concentration of solution can be expressed in various ways as discussed below.

It refers to the amount of the solute per 100 parts if the solution. It can also be called as parts per hundred (pph). It can be expressed by any of following four methods:

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(i) **Weight by weight percentage (%w/w)** = $\frac{\text{Wt. of solute (g)}}{\text{Wt. of solution (g)}} \times 100$

e.g., 10%Na_2CO_3 solution w/w means 10 g of Na_2CO_3 is dissolved in 100 g of the solution. (It means 10 g Na_2CO_3 is dissolved in 90 of solvent)

(ii) **Weight by volume percent (%w/v)** = $\frac{\text{Wt. of solute (g)}}{\text{Wt. of solution (cm}^3\text{)}} \times 100$

e.g., 10%Na_2CO_3 (w/v) means 10 g Na_2CO_3 is dissolved in 100 cm^3 of solution

(iii) **Volume by volume percent (%v/v)** = $\frac{\text{Volume of solute (cm}^3\text{)}}{\text{Volume of solution (cm}^3\text{)}} \times 100$

e.g.,10% ethanol (v/v) means 10 cm^3 of ethanol dissolved in 100 cm^3 of solution.

(iv) **Volume by volume percent (%v/v)** = $\frac{\text{Vol. of solute}}{\text{Wt. of solution}} \times 100$

e.g.,10% ethanol (v/w) means 10 cm^3 of ethanol dissolved in 100 g of solution.

Example Concentrated nitric acid used as laboratory reagent is usually 69% by mass of nitric acid. Calculate the volume of the solution which contains 23 g nitric acid. The density of concentrated acid is 1.41 g cm^{-3}.

Solution 69 g of HNO_3 in 100 g solution

given density = 1.41 g/cc

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{volume}} = \frac{10000}{1.41}\text{ cc}$$

Now,

69 g HNO_3 is in $\frac{10000}{141}$ volume solution

23 g HNO_3 $\rightarrow$ $\frac{23}{69} \times \frac{10000}{141}$ volume solution = **23.64 cm^3**

22. Molarity (M)

The number of moles of solute dissolved in one litre solution is called its molarity.

$$\text{Molarity} = \frac{\text{Number of moles of solute}}{\text{volume of solution in litres}} = \frac{n}{V}$$

$$\text{weight of solute in gram} = \frac{\text{Molarity} \times \text{Volume of solution in mL} \times \text{molecular weight}}{1000}$$

$$\text{Molarity} = \frac{\text{Numbers of moles of solute} \times 1000}{\text{Volume of solution in mL}}$$

$$= \frac{\text{weight of solute in grams} \times 1000}{\text{Molecular weight} \times \text{Volume of solution in mL}}$$

$$= \frac{\text{strength of solution in gram/litre}}{\text{Molecular weight of solute}} = \text{Normality} \times \frac{\text{Equivalent weight of solute}}{\text{Molecular weight of solute}}.$$

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Example A bottle of commercial sulphuric acid (density 1.787 g ml^{-1}) is labelled as 86% by weight. What is the molarity of acid?

Solution 86 g H_2SO_4 is in 100 g solution

Now

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$1.78 = \frac{100}{V}$$

$$V = \frac{10000}{1.78} = 56.18 \text{ ml.}$$

Again

$$\text{molarity(M)} = \frac{\text{no. of moles of } H_2SO_4 \times 1000}{\text{Volume of sol. in ml.}}$$

$$= \frac{\left(\frac{86}{98}\right) \times 1000}{56.18} = \mathbf{15.62\ M}$$

23. Molarity (m)

The number of moles or gram molecules of solute dissolved in 1000 gram of the solvent is called molality of the solution.

$$\text{Molality of a solution} = \frac{\text{Number of moles of solute}}{\text{Amount of solvent in kg}} = \frac{\text{Number of moles of solute} \times 1000}{\text{Amount of solved in gram}}$$

It is independent of temperature.

24. Parts per million (ppm) and parts per billion (ppb)

When a solute is present in very small quantity, it is convenient to express the concentration in parts per million and parts per billion. It is the number of parts of solute per million (10^6) or per billion (10^9) parts of solution. It is independent of the temperature.

$$\text{ppm} = \frac{\text{Mass of solute component}}{}$$

$$\text{ppb} = \frac{\text{Mass of solute component}}{}$$

Example Calculate the parts per million of SO_2 gas in 250 ml water (density 1g cm^{-3}) containing 5×10^{-4} g of SO_2 gas.

Solution Mass of SO_2 gas = 5×10^{-4} g; Mass of H_2O = Volume×Density = 250 cm^3×1g cm^3 = 250 g

$$\therefore \text{Parts per million of } SO_2 \text{ gas} = \frac{5\times10^{-4}}{250\text{g}} \times 10^6 = \mathbf{2}$$

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25. Formality (F)

Formality of solution may be defined as the number of gram formula units of the ionic solute dissolved per litre of the solution. It is represented by **F.** Commonly, the term formality is used to express the concentration of the ionic solids which do not exist as molecules but exist as network of ions. A solution containing one gram formula mass of solute per liter of the solution has formality equal to one and os called **Formal solution.** It may be mentioned here that the formality of a solution changes with change in temperature.

$$\text{Formality(F)} = \frac{\text{Number of gram formula units of solute}}{\text{Volume of solution in litres}}$$

$$= \frac{\text{Mass of ionic solute (g)}}{\text{gram formula unit mass of solute} \times \text{Volume of solution (l)}}$$

Example What will be the formality of KNO_3 solution having strength equal to 2.02 g per litre?

Solution Strength of KNO_3 =2.02 gL^{-1} and g formula weight of KNO_3 =101 g

$$\therefore \text{Formality of } KNO_3 = \frac{\text{strength in } gl^{-1}}{\text{g. formula wt. of } KNO_3} = \frac{2.02}{101} = \mathbf{0.02F}$$

26. Mole fraction (x)

The ratio of moles of one component to the total number of moles of all the components present in the solution, is called the mole fraction of that component.

Mole fraction of solute X_A is given by $X_A = \frac{n_A}{n_A + n_B}$

Mole fraction of solute X_B is given by $X_B = \frac{n_B}{n_A + n_B}$

where n_A is moles of solute A and n_B is moles of solvent B.

27. Mass Fraction

Mass fraction of a component in a solution is the mass of the component divided by the total mass of the solution. For a solution containing w_A gm of A and w_B gm of B.

Mass fraction of A= $\frac{W_A}{W_A+W_B}$ Mass fraction of B= $\frac{W_B}{W_A+W_B}$

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NOTE ☞ It may be notes that molarity, mole fraction, mass fraction etc. are preferred to molarity, normality, formality etc. Because the former involve the weights of the solute and solvent where as later involve volumes of solutions. Temperature has no effect on weights but it has significant effect on volumes.

28. Equivalent Weight

Equivalent weight of an element is that part by weight which combines with 1.0 g of hydrogen or 8.0 g of oxygen or 35.5 g of chlorine.

(i) Equivalent weight of a salt (EW) $= \dfrac{\text{Molar mass}}{\text{Net positive (or nagative) valency}}$

e.g. Equivalent weight $CaCl_2 = \dfrac{M}{2}$, $Alcl_3 = \dfrac{M}{3}$, $Al_2(So_4)_3 = \dfrac{M}{6}$

(ii) Equivalent weight of acids $= \dfrac{\text{Moller mass}}{\text{Basicity}}$

e.g. Equivalent weight $HCl = M(\text{basicity} = 1)$; $H_2SO_4 = \dfrac{M}{2}$ (basicity = 2)

$H_3PO_4 = \dfrac{M}{3}$ (basicity = 3)

(iii) Equivalent weight of bases $= \dfrac{\text{Moller mass}}{\text{Acidity}}$

e.g. Equivalent weight $NaOH = M$, $Ca(OH)_2 = \dfrac{M}{2}$, $Al(OH)_3 = \dfrac{M}{3}$

The number of gram-equivalents (Eq)

$$\text{Equivalent} = \frac{\text{Weight of compound}}{\equiv \text{valent weight}} = \frac{w}{\text{Equivalent weight}}$$

Mole Equivalent Relationship In a given weight (w) of sample, number of moles (n) and number of equivalents (eq) are related as

$$n = \frac{w}{m} \text{ and } Eq = \frac{w}{\text{Equivalent weight}}$$

$$\boxed{\frac{Eq}{n} = \frac{M}{\text{Equivalent weight}} = n - \text{factor}}$$

n-factor For salt, it is valency, for acid it is basicity, for base it is acidity.

Normally/Molarity Relationship

$$N = \frac{Eq}{V} \text{ and } M = \frac{n}{V} \Rightarrow \frac{N}{M} = \frac{Eq}{n} = \frac{MW}{EW} = n - \text{factor}$$

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29. Relation Between Molarity And Normality

S = Molarity × molecular weight of solute and S = Normality × equivalent weight of solute.

So we can write

Molarity × molecular weight of solute = Normality × equivalent weight of solute.

$$\text{Normality} = \frac{\text{molarity} \times \text{molecular weight of solute}}{\text{equivalent weight of solute}} = \frac{\text{molarity} \times \text{molecular weight of solute}}{(\text{moleculer weight of solute}/\text{valency factor}}$$

Normality = molarity × valency factor

$N = M \times n$; $N > M$

Example Calculate the molarity and molality of a solution of H_2SO_4 (sp. gr.=1.98) containing 27% H_2SO_4 by mass.
3.3 M, 3.77 M

Solution Vol of 100 g of 27% H_2SO_4 = $\frac{\text{wt.}}{\text{d}} = \frac{100}{1.098}$ ml

$$M_{H_2SO_4} = \frac{\text{wt./mol.wt.}}{\text{vol, of solution (litre)}} = \frac{27 \times 1.198 \times 1000}{98 \times 100} = \mathbf{3.3 mol\ L^{-1}}$$

$$M_{H_2SO_4} = \frac{\text{wt./mol.wt.}}{\text{vol, of solvent (kg)}} = \frac{27 \times 1000}{(100-27) \times 98} = \mathbf{3.77 mol\ Kg^{-1}}$$

30. Dilution Formula

If a concentrated solution is diluted, following formula work

$M_1 V_1 = M_2 V_2$

(M_1 and V_1 are the molarity and volumes before dilution and M_2 and V_2 are molarity and volumes after dilution)

31. Mixing of two or more solutions of different molarities

If two or more solutions of molarities $(M_1, M_2, M_3, \ldots)$ are mixed together, molarity of the resulting solution can be worked out as :

$$M = \frac{M_1 V_1 + M_2 V_2 + M_3 V_3 \ldots}{V_1 + V_2 + V_3 \ldots}$$

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32. Strength of H_2O_2 solution

The strength of H_2O_2 is aqueous solution is expressed in the following two ways:

(i) Percentage strength

The mass of H_2O_2 present in 100 ml of the aqueous solution is termed as percentage strength. For example, a 25% solution (w/v) of H_2O_2 means that 25 grams of H_2O_2 are present in 100 ml of the solution.

(ii) Volume strength

Strength of the sample of H_2O_2 is generally expressed in terms of the volume of oxygen at 0°C and 1 atm that one volume of the sample of hydrogen peroxide gives on heating. The commercial samples are marked as '10 volume'. '15 volume' or '20 volume'. 10 volume means that one volume of the sample of hydrogen peroxide gives 10 volumes of oxygen at 0°C and 1 atm. 1 ml of a 10 volume solution of H_2O_2 will liberate 10 ml of oxygen at 0°C and 1 atm.

33. Percentage labelling of oleum

Oleum is fuming sulphuric acid which contains extra SO_3 dissolved in H_2SO_4 . To convert this extra SO_3 into H_2SO_4, water has to be added ($SO_3+H_2O \rightarrow H_2SO_4$). The amount of sulphuric acid obtained when just sufficient water is added into 100 g of oleum so that all SO_3 present in it is converted into H_2SO_4 is called percentage labelling of oleum.

34. Relationship Between Different Concentration Terms

(i) $N = M \times n$ factor

(ii) $M = \dfrac{md}{1 + mM_2/1000}$

(iii) $m = \dfrac{1000 \times x_2}{x_1 M_1}$

(iv) $M = \dfrac{1000 \times dx_2}{x_1 M_1 + x_1 M_1}$

(v) $d = M\left(\dfrac{1}{m} + \dfrac{M_2}{1000}\right)$

(vi) Volume strength of $H_2O_2 = 5.6 \times N = \dfrac{5.6 \times \text{Percentage strength}}{\text{Eq. wt. of } H_2O_2 (17)} \times 10$

(vii) Volume strength of $H_2O_2 = 11.2 \times M = \dfrac{11.2 \times \text{Percentage strength} \times 10}{\text{Mol. wt. of } H_2O_2 (34)}$

(viii) In oleum labelled as $(100 + x)\%$

$\%$ of free $SO_3 = \left(\dfrac{80 \times x}{18}\right)$ (w/w)

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where N=Normality

M =	Molarity	m =	molarity
d =	density of solution	M_2=	Molecular mass of solute
x_2 =	Mole fraction of solute	x_1 =	Mole fraction of solvent
M_1=	Molecular mass of solvent	d =	Density of solution

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JEE Main Pattern
Exercise (1)

Q1. The ratio mass of oxygen and nitrogen of a particular gaseous mixture is 1:4. The ratio of number of their molecule is
(a) 1:4
(b) 7:32
(c) 1:8
(d) 3:16

Q2. The molarity of a solution obtained by mixing 750 mL of 0.5 M HCl with 250 mL of 2 M HCl will be
(a) 0.875
(b) 1.00M
(c) 1.75M
(d) 0.0975M

Q3. The normality of 0.3 M phosphorus acid (H_3PO_3) is
(a) 0.1
(b) 0.9
(c) 0.3
(d) 0.6

Q4. Dissolving 120 g of urea (mol. wt. 60) in 1000g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
(a) 1.78M
(b) 2.00M
(c) 2.05M
(d) 2.22M

Q5. The mass of Mg_3N_2 produced if 48 gm of Mg metal is reacted with 34 gm NH_3 gas is

$$Mg + NH_3 \rightarrow Mg_3N_2 + H_2$$

(a) $\frac{200}{3}$
(b) $\frac{100}{3}$
(c) $\frac{400}{3}$
(d) $\frac{150}{3}$

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Q6. What is the maximum amount of nitrogen dioxide that can be produced by mixing 4.2 gm of NO(g) and 3.2 gm of O_2(g)?
(a) 4.60g
(b) 2.30g
(c) 3.22g
(d) 6.44g

Q7. Which has maximum moles number of atoms?
(a) 24g of C(12)
(b) 56g of Fe(56)
(c) 27g of Al(27)
(d) 108g of Ag(108)

Q8. Equal volumes of 10% (v/v) of HCl solution. If density of pure NaOH is 1.5 times that of pure HCl then the resultant solution be.
(a) basic
(b) neutral
(c) acidic
(d) can't be predicted.

Q9. Given that the abundances of isotopes $_{54}Fe$, $_{56}Fe$ and $_{57}Fe$ are 5%, 90% and 5%, respectively, the atomic mass of Fe is
(a) 55.85
(b) 55.95
(c) 55.75
(d) 56.05

Q10. 100 mL of 30% (w/v) NaOH solution is mixed with 100 mL 90% (w/v) NaOH solution. Find the molarity of final solution.
(a) 1.3
(b) 13
(c) 1/5
(d) 15

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(b)	(a)	(d)	(c)	(a)
Q6	Q7	Q8	Q9	Q10
(d)	(a)	(a)	(b)	(d)

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JEE Advanced Pattern
Exercise (1)

Q1. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
(a) 1.78M
(b) 2.00M
(c) 2.05M
(d) 2.22M

Paragraph question
$FeSO_4$ undergoes decomposition as
$2FeSO_4(s) \rightarrow Fe_2O_3(s)+SO_2(g)+SO_3(g)$
At 1 atm & 273 K if (7.6 gm) $FeSO_4$ is taken then.

Q2. The volume occupied by the gases at 1 atm & 273K.
(a) 22.4 lit
(b) 11.2 lit
(c) 1.12 lit
(d) 2.24 lit

Q3. The average molar mass of the gaseous mixture.
(a) 72
(b) 36
(c) 48
(d) 60

Assertion and Reason
(a) Both Statement I and Statement II are correct; Statement II is the correct explanation of Statement I
(b) Both Statement I and Statement II are correct; Statement II is not the correct explanation of Statement I
(c) Statement I is correct; Statement II is incorrect
(d) Statement I is incorrect; Statement II is correct

Q4. **Statement I :**Molality of pure ethanol is lesser than pure water.
Statement II : As density of ethanol is lesser than density of water.
[Given : $d_{ethanol}$ = 0.789 gm/ml; d_{water} = 1 gm/ml]

Q5. **Statement I :** A one molal solution prepared at 20°C will retain the same molality at 100°C, provided there is no loss of solute or solvent on heating.
Statement II : Molality is independent of temperatures.

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Q6. A mixture of C_3H_8 (g) O_2 having total volume 100 ml in an Eudiometry tube is sparked & it is observed that a contraction of 45 ml is observed what can be the composition of reacting mixture.
(a) 15 ml C_3H_8 & 85 ml O_2
(b) 25 ml C_3H_8 & 75 ml O_2
(c) 45 ml C_3H_8 & 55 ml O_2
(d) 55 ml C_3H_8 & 45 ml O_2

Q7. Which has the maximum number of atoms :
(a) 24 g C
(b) 56 g Fe
(c) 27 g Al
(d) 108 g Ag (108)

Q8. Match the column:

Column I	Column II
(A) 16 g of CH_4	(P) 1/2 mole molecule
(B) 1 g of H_2	(Q) $6.023\times10^{23}\times5$ atoms
(C) 22 g of CO_2	(R) 11.2 litre
(D) 9 g of H_2O	(S) 1.806×10^{23} atoms

Q9. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution
(a) 0.100
(b) 0.190
(c) 0.086
(d) 0.050

Q10. A compound H_2X with molar weight of 80 g is dissolved in a solvent having density of 0.4 g mL^{-1}. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(c)	(c)	(a)	(b)	(a)
Q6	Q7	Q8	Q9	Q10
(a), (b)	(a)	(A) - Q (B) - P , R (C) - P , R (D) - P	(c)	8

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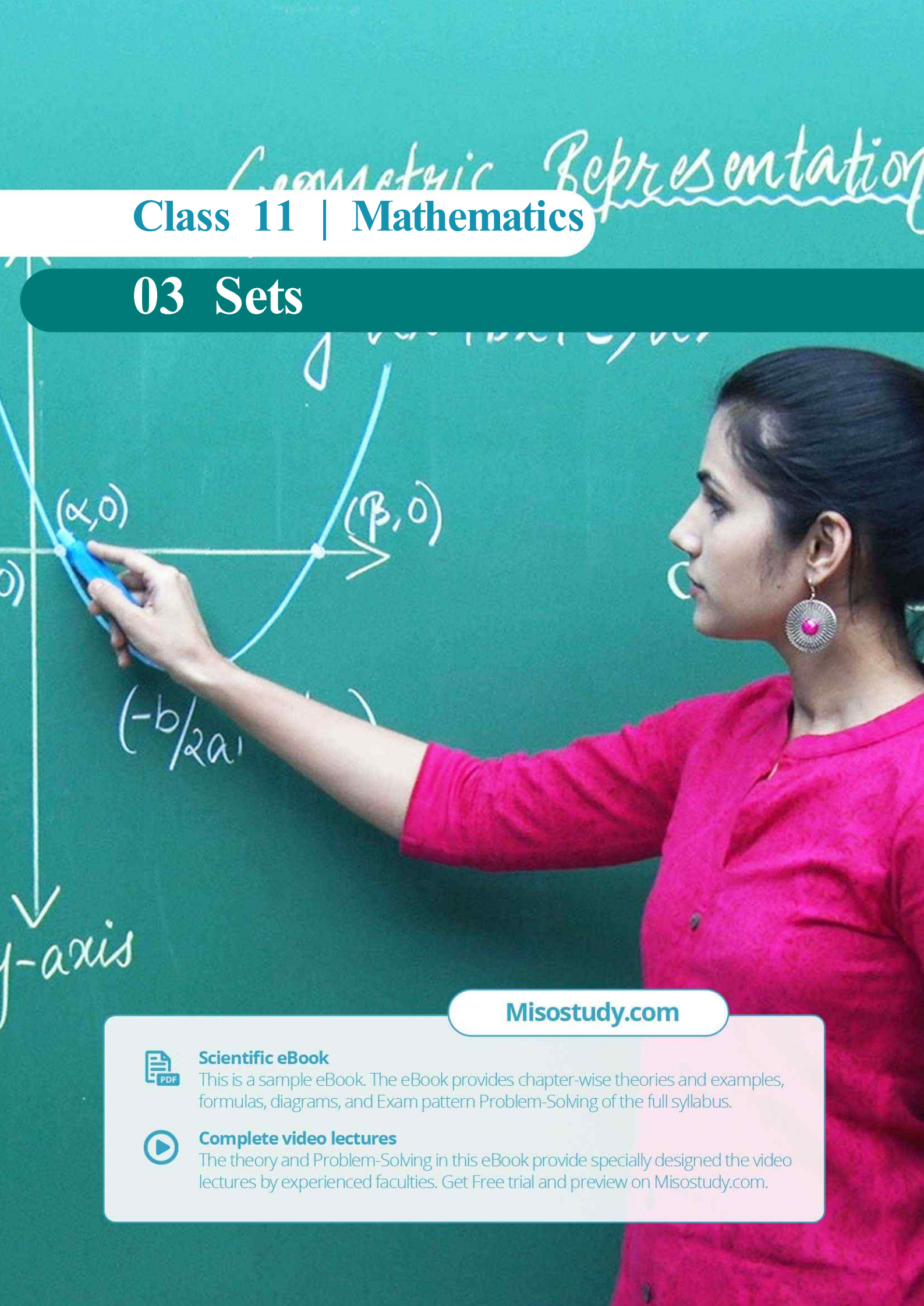
03 Sets



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01. Definition of Set

Set as "a well defined collection of objects".

Example The collection of vowels in English alphabets. This set contains five elements, namely, a, e, i, o, u.

NOTE ☞ Collection of good teachers in a school is not a set.

02. Reserved Symbols

We reserve some symbols for these set:

① **N** : for the set of natural numbers.
② **Z** : for the set of integers.
③ $\mathbf{Z}^+$: for the set of all positive integers.
④ **Q** : for the set of all rational numbers.
⑤ $\mathbf{Q}^+$: for the set of all positive rational numbers.
⑥ **R** : for the set of all real numbers.
⑦ $\mathbf{R}^+$: for the set of all positive real numbers.
⑧ **C** : for the set of all complex numbers.

03. Description of a Set

A set is often described in the following two forms. One can make use of any one of these two ways according to his (her) convenience.

(i) Roster form or Tabular form
(ii) Set-builder form

ROSTER FORM

In this form a set is described by listing elements, separated by commas, within braces {}.

NOTE ☞ (1) The order in which the elements are written in a set makes no difference.
(2) Also, the repetition of an element has no effect.

SET-BUILDER FORM

In this form, a set is described by a characterizing property $P(x)$ of its elements x. In such a case the set is described by $\{x : P(x) \text{holds}\}$ or, $\{x | P(x) \text{holds}\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol $'|'$ or $':'$ is read as 'such that'.

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04. Types of Sets

(1) EMPTY SET A set is said to be empty or null or void set if it has no element and it is denoted by ϕ(phi).
In Roster method, ϕ is denoted by $\{\}$.

(2) SINGLETON SET A set consisting of a single element is called a singleton set.

(3) FINITE SET A set is called a finite set if it is either void set or its elements can be listed (counted, labelled) by natural numbers $1, 2, 3, \ldots$ and the process of listing terminates at a certain natural number n (say).

(4) CARDINAL NUMBER OF A FINITE SET The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by $n(A)$.

(5) INFINITE SET A set whose elements cannot be listed by the natural numbers $1, 2, 3, \ldots, n$, for any natural number n is called an infinite set.

(6) EQUIVALENT SETS Two finite sets A and B are equivalent if their cardinal numbers are some i.e. $n(A) = n(B)$.

(7) EQUAL SETS Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

05. Subsets

Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B.
If A is a subset of B, we write $A \subseteq B$, which is read as "A is a subset of B" or "A is contained in B".
Thus, $A \subseteq B$ if $a \in A \Rightarrow a \in B$.

SOME RESULTS ON SUBSETS

RESULT 1 Every set is a subset of itself.

RESULT 2 The empty set is a subset of every set.

RESULT 3 The total number of subsets of a finite set containing n elements is 2^n.

REMARK SUBSETS OF THE SET **R** OF REAL NUMBERS

i) The set of all natural numbers $N = \{1, 2, 3, 4, 5, 6, \ldots\}$

ii) The set of all integers $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

iii) The set of all rational numbers $Q = \{x : x = \frac{m}{n}, m, n \in Z, n \neq 0\}$.

iv) The set of all irrational numbers. It is denoted by T.

Thus,
$T = \{x : x \in R \text{ and } x \notin Q\}$
Clearly, $N \subset Z \subset Q \subset R$, $T \subset R$ and $N \not\subset T$.

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06. Universal Set

A set that contains all sets in a given context is called the universal set.

Example If $A=\{1,2,3\}$, $B=\{2,4,5,6\}$ and $C=\{1,3,5,7\}$, then $U=\{1,2,3,4,5,6,7\}$ can be taken as the universal set.

07. Power Set

Let A be a set. Then the collection or family of all subsets of A is called the power set of A and is denoted by P(A). The power set of a given set is always non-empty.

Example Let $A=\{1,2,3\}$. Then, the subsets of A are:

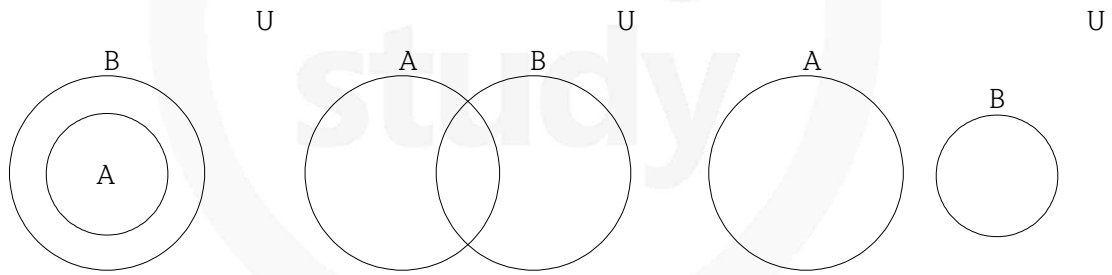
$\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$, and $\{1,2,3\}$.

Hence, $P(A)=\{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

08. Venn Diagrams

In Venn-diagrams the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle.

A set containing other set **Intersecting sets** **Non intersecting sets**



09. Operations On Sets

(1) **UNION OF SETS**

Let A and B be two sets. The union of A and B is the set of all those elements which belong either to A or to B or to both A and B.

We shall use the notation $A \cup B$ (read as "A union B") to denote the union of A and B.

Thus, $A \cup B=\{x : x \in A \text{ or } x \in B\}$.

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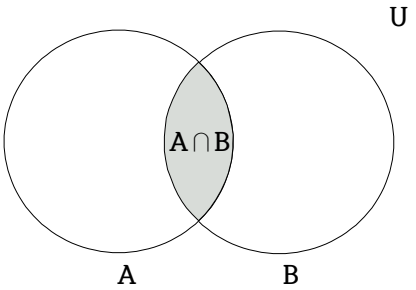
(2) **INTERSECTION OF SETS**

Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B.

The intersection of A and B is denoted by $A \cap B$ (read as "A intersection B")

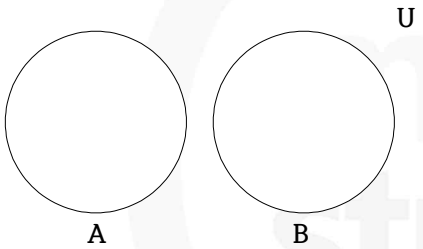
Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.



(3) **DISJOINT SETS**

Two sets A and B are said to be disjoint, if $A \cap B = \phi$.

If $A \cap B \neq \phi$, then A and B said to be intersecting or overlapping sets.

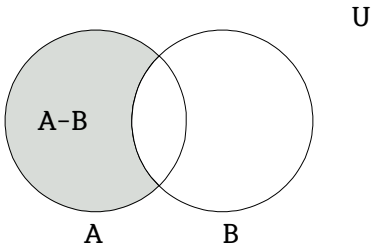


(4) **DIFFERENCE OF SETS**

Let A and B be two sets. The difference of A and B, written as $A - B$, is the set of all those elements of A which do not belong to B.

Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

or, $A - B = \{x \in A : x \notin B\}$



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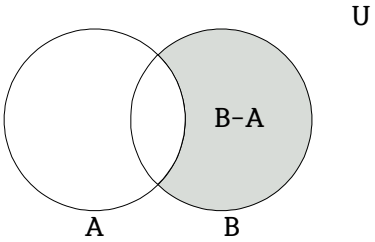
Similarly, the difference $B-A$ is the set of all those elements of B that do not belong to A i.e.

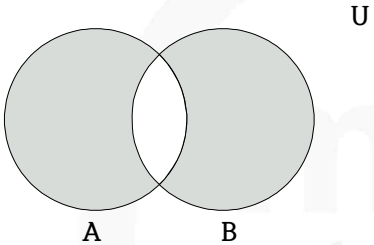
$$B-A=\{x\in B: x\notin A\}.$$



(5) **SYMMETRIC DIFFERENCE OF TWO SETS**

Let A and B be two sets. The symmetric difference of sets A and B is the set $(A-B)\cup(B-A)$ and is denoted by $A\triangle B$.

Thus, $A\triangle B=(A-B)\cup(B-A)=\{x: x\notin A\cap B\}$

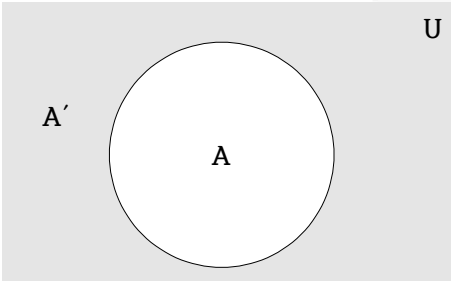


(6) **COMPLEMENT OF A SET**

Let U be the universal set and let A be a set such that $A\subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $U-A$ and is defined the set of all those elements of U which are not in A.

Thus $\quad A'=\{x\in U: x\notin A\}.$

Clearly, $\quad x\in A' \Leftrightarrow x\notin A.$



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10. Laws of Algebra of Sets

RESULT 1 (Idempotent Laws) For any set A, we have
(i) $A \cup A = A$

PROOF
$A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$
$A \cap A = A.$
$A \cap A = \{x : x \in A \text{ and } x \in A\} = \{x : x \in A\} = A.$

RESULT 2 (identity Laws) For any set A, we have
(i) $A \cup \phi = A$
i.e. ϕ and U are identity elements for union and intersection respectively.

PROOF
$A \cup \phi = \{x : x \in A \text{ or } x \in \phi\} = \{x : x \in A\} = A$
(ii) $A \cap U = A.$
$A \cap U = \{x : x \in \text{ and } x \in U\} = \{x : x \in A\} = A$

RESULT 3 (Commutative Laws) For any two sets A and B, we have
(i) $A \cup B = B \cup A$
i.e. union and intersection are commutative.

PROOF
Recall that two sets X and Y are equal iff $X \subseteq Y$ and $Y \subseteq X$. Also, $X \subseteq Y$ if every element of X belongs to Y.
Let x be an arbitrary element of $A \cup B$. Then,

$$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B \Rightarrow x \in B \text{ or } x \in A \Rightarrow x \in B \cup A$$

$\therefore \quad A \cup B \subseteq B \cup A.$
Similarly, $B \cup A \subseteq A \cup B.$
Hence, $\quad A \cup B = B \cup A.$
(ii) $A \cap B = B \cap A$
Let x be an arbitrary element of $A \cap B$.
Then, $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$$\Rightarrow x \in B \text{ and } x \in A \Rightarrow x \in B \cap A$$

$\therefore \quad A \cap B \subseteq B \cap A$
Similarly, $B \cap A \subseteq A \cap B$
Hence, $\quad A \cap B = B \cap A.$

RESULT 4 (Associative Laws) If A, B and C are any three sets, then
(i) $(A \cup B) \cup C = A \cup (B \cup C)$
i.e. union and intersection are associative.

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PROOF

Let x be an arbitrary element of $(A \cup B) \cup C$. Then,

$x \in (A \cup B) \cup C$

$\Rightarrow$ $x \in (A \cup B)$ or $x \in C$

$\Rightarrow$ $(x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow$ $x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow$ $x \in A$ or $x \in (B \cup C)$

$\Rightarrow$ $x \in A \cup (B \cup C)$

$\therefore$ $(A \cup B) \cup C \subseteq A \cup (B \cup C)$.

Similarly, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$.

Hence, $(A \cup B) \cup C = A \cup (B \cup C)$.

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Let x be an arbitrary element of $A \cup (B \cap C)$. Then,

$x \in A \cap (B \cap C)$

$\Rightarrow$ $x \in A$ and $x \in (B \cap C)$

$\Rightarrow$ $x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow$ $(x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow$ $x \in (A \cap B)$ and $x \in C$

$\Rightarrow$ $x \in (A \cap B) \cap C$

$\therefore$ $A \cap (B \cap C) \subseteq (A \cap B) \cap C$.

Similarly, $(A \cap B) \cap C \subseteq A \cap (B \cap C)$.

Hence, $A \cap (B \cap C) = (A \cap B) \cap C$.

RESULT 5 (Distributive Laws) If A, B and C are any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

i.e. union and intersection are distributive over intersection and union respectively.

PROOF

Let x be an arbitrary element of $A \cup (B \cap C)$. Then,

$x \in A \cup (B \cap C)$

$\Rightarrow$ $x \in A$ or $x \in (B \cap C)$

$\Rightarrow$ $x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow$ $(x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$ [$\because$ 'or' is distributive over 'and']

$\Rightarrow$ $x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow$ $x \in ((A \cup B) \cap (A \cup C))$

$\therefore$ $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Similarly, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

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(ii) $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$.

Let x be an arbitrary element of $A \cap (B \cup C)$. Then,

$x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ and $x \in (B \cup C)$

$\Rightarrow$ $x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow$ $(x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow$ $x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow$ $x \in (A \cap B) \cup (A \cap C)$

$\therefore$ $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Similarly, $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

RESULT 6 (De-morgan's Laws) If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

PROOF

Let x be an arbitrary element of $(A \cup B)'$. Then,

$x \in (A \cup B)'$

$\Rightarrow$ $x \notin (A \cup B)$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \in A' \cap B'$.

$\therefore$ $(A \cup B)' \subseteq A' \cap B'$.

Again, let y be an arbitrary element of $A' \cap B'$. Then,

$y \in A' \cap B'$

$\Rightarrow$ $y \in A'$ and $y \in B'$

$\Rightarrow$ $y \notin A$ and $y \notin B$

$\Rightarrow$ $y \notin A \cup B$.

$\Rightarrow$ $y \in (A \cup B)'$

$\therefore$ $A' \cap B' \subseteq (A \cup B)'$.

Hence, $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$.

Let x be an arbitrary element of $(A \cap B)'$. Then,

$x \in (A \cap B)'$

$\Rightarrow$ $x \notin (A \cap B)$

$\Rightarrow$ $x \notin A$ or $x \notin B$

$\Rightarrow$ $x \in A'$ or $x \in B'$

$\Rightarrow$ $x \in A' \cup B'$

$\Rightarrow$ $(A \cap B)' \subseteq A' \cup B'$.

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Again, let y be an arbitrary element of $A' \cup B'$. Then,

$y \in (A' \cap B')$

$\Rightarrow \quad y \in A'$ or $y \in B'$

$\Rightarrow \quad y \notin A$ or $y \notin B$

$\Rightarrow \quad y \notin (A \cap B)$

$\Rightarrow \quad y \in (A \cap B)'$

$\therefore \quad A' \cup B' \subseteq (A \cap B)'$.

Hence, $\quad (A \cap B)' = A' \cup B'$.

11. MORE RESULTS ON OPERATIONS ON SETS

RESULT 1 If A and B are any two sets, then

(i) $\quad A - B = A \cap B'$

PROOF

Let x be an arbitrary element of $A - B$. Then,

$x \in (A - B)$

$\Rightarrow \quad x \in A$ and $x \notin B$

$\Rightarrow \quad x \in A$ and $x \notin B'$

$\Rightarrow \quad x \in A \cap B'$

$\therefore \quad A - B \subseteq A \cap B'$ ...(i)

Again, let y be an arbitrary element of $A \cap B'$. Then,

$y \in A \cap B'$

$\Rightarrow \quad y \in A$ and $y \in B'$

$\Rightarrow \quad y \in A$ and $y \notin B$

$\Rightarrow \quad y \in A - B$

$\therefore \quad A \cap B' \subseteq (A - B)$ ...(ii)

Hence, from (i) and (ii), we have $A - B = A \cap B'$.

(ii) $\quad B - A = B \cap A'$

Let x be an arbitrary element of $B - A$. Then,

$x \in B - A$

$\Rightarrow \quad x \in B$ and $x \notin A$

$\Rightarrow \quad x \in B$ and $x \in A^c$

$\Rightarrow \quad x \in B \cap A^c$

$\therefore \quad B - A \subseteq B \cap A^c$ ...(i)

Again, let y be an arbitrary element of $B \cap A^c$

$\Rightarrow \quad y \in B$ and $y \in A^c$

$\Rightarrow \quad y \in B$ and $y \notin A$

$\Rightarrow \quad y \in B - A$

$\therefore \quad A \cap B^c \subseteq (A - B)$ ...(ii)

Hence, from (i) and (ii), we have $A - B = A \cap B^c$

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(iii) $A-B=A \Leftrightarrow A\cap B=\phi$

In order to prove that $A-B=A \Leftrightarrow A\cap B=\phi$

we shall prove that:

(i) $A-B=A \Rightarrow A\cap B=\phi$,

(ii) $A\cap B=\phi \Rightarrow A-B=A$.

First, let $A-B=A$. Then we have to prove that $A\cap B=\phi$. If possible, let $A\cap B\neq\phi$. Then,

$$A\cap B\neq\phi \Rightarrow \text{there exists } x\in A\cap B$$
$$\Rightarrow x\in A \text{ and } x\in B \Rightarrow x\in A-B \text{ and } x\in B \qquad [\because A-B=A]$$
$$\Rightarrow (x\in A \text{ and } x\notin B) \text{ and } x\in B \qquad [\text{By def. of } A-B]$$
$$\Rightarrow x\in A \text{ and } (x\notin B \text{ and } x\in B)$$

But $x\notin B$ and $x\in B$ both can never be possible simultaneously. Thus, we arriver at a contradiction, So, our supposition is wrong.

$\therefore \quad A\cap B=\phi$

Hence, $\quad A-B=A \Rightarrow A\cap B=\phi \qquad \ldots(i)$

Conversely, let $A\cap B=\phi$. Then we have to prove that $A-B=A$. For this we shall show that $A-B\subseteq A$ and $A\subseteq A-B$

Let x be an arbitrary element of $A-B$. Then,

$$x\in A-B \Rightarrow x\in A \text{ and } x\notin B$$
$$\Rightarrow x\in A$$

$\therefore \quad A-B\subseteq A.$

Again let y be an arbitrary element of A. Then,

$$y\in A \Rightarrow y\in A \text{ and } y\notin B \qquad [\because A\cap B=\phi]$$
$$\Rightarrow y\in A-B \qquad [\text{By def. of } A-B]$$

$\therefore \quad A\subseteq A-B.$

So, we have $A-B\subseteq A$ and $A\subseteq A-B$. Therefore, $A-B=A$.

Thus, $A\cap B=\phi \Rightarrow A-B=A \qquad \ldots(i)$

Hence, from (i) and (ii), we have

$$A-B=A \Leftrightarrow A\cap B=\phi.$$

(iv) $(A-B)\cup B=A\cup B$

Let x be an arbitrary element of $(A-B)\cup B$. Then,

$$x\in(A-B)\cup B$$
$$\Rightarrow \quad x\in A-B \text{ or } x\in B$$
$$\Rightarrow \quad (x\in A \text{ and } x\notin B) \text{ or } x\in B$$
$$\Rightarrow \quad (x\in A \text{ or } x\in B) \text{ and } (x\notin B \text{ or } x\in B)$$
$$\Rightarrow \quad x\in A\cup B$$
$$\therefore \quad (A-B)\cup B\subseteq A\cup B$$

Let y be an arbitrary element of $A\cup B$. Then,

$$y\in A\cup B$$
$$\Rightarrow \quad y\in A \text{ or } y\in B$$
$$\Rightarrow \quad (y\in A \text{ or } y\in B) \text{ and } (y\notin B \text{ or } y\in B)$$
$$\Rightarrow \quad (y\in A \text{ and } y\notin B) \text{ or } y\in B$$
$$\Rightarrow \quad y\in(A-B)\cup B$$
$$\therefore \quad A\cup B\subseteq(A-B)\cup B$$

Hence, $\quad (A-B)\cup B=A\cup B.$

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(v) $(A-B)\cap B=\phi$

If possible let $(A-B)\cap B\neq\phi$. Then, there exists at least one element x, (say), in $(A-B)\cap B$.

Now, $x\in(A-B)\cap B \Rightarrow x\in(A-B)$ and $x\in B$

$\Rightarrow (x\in A$ and $x\notin B)$ and $x\in B$

$\Rightarrow x\in A$ and $(x\notin B$ and $x\in B)$

But, $x\notin B$ and $x\in B$ both can never be possible simultaneously. Thus, we arrive at a contradiction. So, our supposition is wrong.

Hence, $(A-B)\cap B=\phi$

(vi) $A\subseteq B\Leftrightarrow B'\subseteq A'$

First, let $A\subseteq B$. Then we have to prove that $B'\subseteq A'$. Let x be an arbitrary element of B'. Then,

$x\in B' \Rightarrow x\notin B$

$\Rightarrow x\notin A$ $\quad[\because A\subseteq B]$

$\Rightarrow x\in A'$

$\therefore$ $B'\subseteq A'$.

Thus, $A\subseteq B\Rightarrow B'\subseteq A'$ ...(i)

Conversely, let $B'\subseteq A'$. Then, we have to prove that $A\subseteq B$. Let y be an arbitrary element of A. Then,

$y\in A \Rightarrow y\notin A'$

$\Rightarrow y\notin B'$ $\quad[\because B'\subseteq A']$.

$\Rightarrow y\in B$

$\therefore$ $A\subseteq B$.

Thus, $B'\subseteq A'\Rightarrow A\subseteq B$ ...(ii)

From (i) and (ii), we have $A\subseteq B\Leftrightarrow B'\subseteq A'$.

(vii) $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$

Let x be an arbitrary element of $(A-B)\cup(B-A)$. Then,

$x\in(A-B)\cup(B-A)$

$\Rightarrow$ $x\in A-B$ or $x\in B-A$

$\Rightarrow$ $(x\in A$ and $x\notin B)$ or $(x\in B$ and $(x\notin A)$

$\Rightarrow$ $(x\in A$ or $x\in B)$ and $(x\notin B$ or $x\notin A)$

$\Rightarrow$ $x\in(A\cup B)$ and $x\notin(A\cap B)$

$\Rightarrow$ $x\in(A\cup B)-(A\cap B)$

$\therefore$ $(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$ ...(i)

Again, let y be an arbitrary element of $(A\cup B)-(A\cap B)$.

Thus, $y\in(A\cup B)-(A\cap B)$

$\Rightarrow$ $y\in A\cup B$ and $y\notin A\cap B$

$\Rightarrow$ $(y\in A$ or $y\in B)$ and $(y\notin A$ and $y\notin B)$

$\Rightarrow$ $(y\in A$ and $y\notin B)$ or $(y\in B$ and $y\notin A)$

$\Rightarrow$ $y\in(A-B)$ or $y\in(B-A)\Rightarrow y\in(A-B)\cup(B-A)$.

$\therefore$ $(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$ ...(ii)

Hence, from (i) and (ii), we have

$(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$.

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RESULT 2 If A, B and C are any three sets, then prove that:
(i) $A-(B\cap C)=(A-B)\cup(A-C)$

PROOF

Let x be any element of $A-(B\cap C)$. Then,

$$\begin{aligned} x\in A-(B\cap C) &\Rightarrow x\in A \text{ and } x\notin(B\cap C)\\ &\Rightarrow x\in A \text{ and } (x\notin B \text{ or } x\notin C)\\ &\Rightarrow (x\in A \text{ and } x\notin B) \text{ or } (x\in A \text{ and } x\notin C)\\ &\Rightarrow x\in(A-B) \text{ or } x\in(A-C)\\ &\Rightarrow x\in(A-B)\cup(A-C)\end{aligned}$$

$\therefore$ $A-(B\cap C)\subseteq(A-B)\cup(A-C)$

Similarly, $(A-B)\cup(A-C)\subseteq A-(B\cap C)$

Hence, $A-(B\cap C)=(A-B)\cup(A-C)$

(ii) $A-(B\cup C)=(A-B)\cap(A-C)$

Let x be an arbitrary element of $A-(B\cup C)$. Then

$$\begin{aligned} x\in A-(B\cup C) &\Rightarrow x\in A \text{ and } x\notin(B\cap C)\\ &\Rightarrow x\in A \text{ and } (x\notin B \text{ and } x\notin C)\\ &\Rightarrow (x\in A \text{ and } x\notin B) \text{ and } (x\in A \text{ and } x\notin C)\\ &\Rightarrow x\in(A-B) \text{ and } x\in A-C\\ &\Rightarrow x\in(A-B)\cap(A-C)\end{aligned}$$

$\therefore$ $A-(B\cup C)\subseteq(A-B)\cap(A-C)$

Similarly, $(A-B)\cap(A-C)\subseteq A-(B\cup C)$

Hence, $A-(B\cup C)=(A-B)\cap(A-C)$

(iii) $A\cap(B-C)=(A\cap B)-(A\cap C)$

Let x be any arbitrary element of $A\cap(B-C)$. Then

$$\begin{aligned} x\in A\cap(B-C) &\Rightarrow x\in A \text{ and } x\in(B-C)\\ &\Rightarrow x\in A \text{ and } (x\in B \text{ and } x\notin C)\\ &\Rightarrow (x\in A \text{ and } x\in B) \text{ and } (x\in A \text{ and } x\notin C)\\ &\Rightarrow x\in(A\cap B) \text{ and } x\notin(A\cap C)\\ &\Rightarrow x\in(A\cap B)-(A\cap C)\end{aligned}$$

$\therefore$ $A\cap(B-C)\subseteq(A\cap B)-(A\cap C)$

Similarly, $(A\cap B)-(A\cap C)\subseteq A\cap(B-C)$

Hence, $A\cap(B-C)=(A\cap B)-(A\cap C)$.

(iv) $A\cap(B\triangle C)=(A\cap B)\triangle(A\cap C)$

$$\begin{aligned} A\cap(B\triangle C) &= A\cap[(B-C)\cup(C-B)]\\ &= [A\cap(B-C)]\cup[A\cap(C-B)] && \text{[By distributive law]}\\ &= [(A\cap B)-(A\cap C)]\cup[(A\cap C)-(A\cap B)] && \text{[Using (iii)]}\\ &= (A\cap B)\triangle(A\cap C)\end{aligned}$$

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12. SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$ i.e. $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \triangle B) =$ No. of elements which belong to exactly one of A or B

$$= n((A - B) \cup (B - A))$$
$$= n(A - B) + n(B - A) \qquad [\because (A - B) \text{ and } (B - A) \text{ are disjoint}]$$
$$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$$
$$= n(A) + n(B) - 2n(A \cap B)$$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) No. of elements in exactly two of the sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3\, n(A \cap B \cap C).$$

(vii) No. of elements in exactly one of the sets A, B, C

$$= n(A) + n(B) + n(C) - 2\, n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$$

(viii) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$.

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JEE Main Pattern
Exercise (1)

Q1. For two events A and B which of the following is simple expression of $(A\cap B)\cup(A\cap B')\cup(A'\cap B)$?
(a) $(A\cap B)$
(b) $(A\cup B)$
(c) $(A'\cap B')$
(d) $(A\cap B')$

Q2. If $U=\{1,2,3\}$ and $A=\{1,2\}$ then $[P(A)]'=$ _____
(a) $\{\{3\},\{2,3\},\{1,3\},\{1,2\},\phi\}$
(b) $\{\{3\},\{2,3\},\{1,3\},\{1,2,3\}\}$
(c) $\{\{3\},\{2,3\},\{1,3\},\{1,2,3\},\phi\}$
(d) $\{\{3\},\{2,3\},\{1,3\},\{1,2\}\}$

Q3. Let U be the universal set and $A\cup B\cup C=U$. Then $[(A-B)\cup(B-C)\cup(C-A)]'$ equals
(a) $A\cup B\cup C$
(b) $A\cap B\cap C$
(c) $A\cup(B\cap C)$
(d) $A\cap(B\cup C)$

Q4. The set $(A\cup B\cup C)\cap(A\cap B'\cap C')'\cap C'$ equals
(a) $B\cap C'$
(b) $B\cup C'$
(c) $A\cap C$
(d) $A\cup C$

Q5. Let $A=\{(x,y): y=e^{x}, x\in R\}$, $B=\{(x,y): y=e^{-x}, x\in R\}$, then
(a) $A\cap B=\phi$
(b) $A\cap B\neq\phi$
(c) $A\cup B=R$
(d) $A\cup B=A$

Q6. Let U be the universal set containing 700 elements. If A, B are sub-sets of U such that $n(A)=200, n(B)=300$ and $n(A\cap B)=100$. Then, $n(A'\cap B')=$
(a) 400
(b) 600
(c) 300
(d) none of these

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Q7. If $A=\{1,2,3,4,5\}$, then the number of proper subset of A is
(a) 120
(b) 30
(c) 31
(d) 32

Q8. If A and B are two sets then $B-(B-A)=$____
(a) $(A-B)-B$
(b) $A-(A-B)$
(c) A
(d) B

Q9. Taking $U=[1,5]$, $A=\{x/x\in \boldsymbol{N}, x^2-6x+5=0\}$, $A'=$___
(a) $\{1,5\}$
(b) $(1,5)$
(c) $[1,5]$
(d) $[-1,-5]$

Q10. If A and B are two sets such that $n(A)=115, n(B)=326, n(A-B)=47$, then write $n(A\cup B)=$
(a) 372
(b) 373
(c) 400
(d) none of these

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(b)	(b), (c), (d)	(b), (c), (d)	(a)	(b)
Q6	Q7	Q8	Q9	Q10
(c)	(c)	(b)	(b)	(b)

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JEE Advanced Pattern
Exercise (1)

Q1. Two finite sets have m and n element respectively. The total number of subsets of first set is 112 more than the total number of subsets of the second set. The value of m and n respectively are:
(a) 5.2
(b) 4.7
(c) 7.4
(d) 2.5

Q2. A survey shows that 70% of the Indians like mango wheres 82% like apple. If x% of Indian like both mango and apples then:
(a) $x = 52$
(b) $52 \le x \le 70$
(c) $x = 70$
(d) $70 \le x \le 82$

Q3. If $X \cup \{3, 4\} = \{1, 2, 3, 4, 5, 6\}$ the which of the following is true
(a) Smallest set $X = \{1, 2, 5, 6\}$
(b) Smallest set $X = \{1, 2, 3, 5, 6\}$
(c) Smallest set $X = \{1, 2, 3, 4\}$
(d) Greatest set $X = \{1, 2, 3, 4\}$

Q4. In a certain town 30% families own a scooter and 40% on a car 50% own neither a scooter nor a car 2000 families own both a scooter and car consider the following statements in this regard

(1) 20% families own both scooter and car
(2) 35% families own either a car or a scooter
(3) 10000 families live in town.

Which of the above statement are correct ?
(a) 2 and 3
(b) 1, 2 and 3
(c) 1 and 2
(d) 1 and 3

Q5. Let $A = \{\theta : \tan\theta + \sec\theta = \sqrt{2}\sec\theta\}$ and $B = \{\theta : \sec\theta - \tan\theta = \sqrt{2}\tan\theta\}$ be two sets then.

(a) $A = B$
(b) $A \subset B$
(c) $A \ne B$
(d) $B \subset A$

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Q6. A d B are two sets $n(A-B)=8+2x$, $n(B-A)=6x$ and $n(A\cap B)=x$. If $n(A)=n(B)$ then $n(A\cap B)=$_____

(a) 26
(b) 50
(c) 24
(d) none of these

Q7. $A=\{(a,b)/b=2a-5\}$ If $(m,5)$ and $(6,n)$ are the member of set A then m and n are respectively

(a) $5,7$
(b) $7,5$
(c) $2,3$
(d) $5,3$

Q8. There are three-three sets given in column-A and column-B

Column-A	Column-B
(1) $\{L,A,T\}$	(A) $\{x/x\in z, x^2<5\}$
(2) $\{x\in z/x^3-x=0\}$	(B) $\{x/x$ is a letter of the word LATA$\}$
(3) $\{-2,-1,0,1,2\}$	(C) $\left\{\sin 0, \sin\frac{3\pi}{2}, \tan\frac{5\pi}{4}\right\}$

Which one of the following matches is correct?

(a) $1-A,\ 2-B,\ 3-C$
(b) $1-B,\ 2-A,\ 3-C$
(c) $1-B,\ 2-C,\ 3-A$
(d) $1-A,\ 2-C,\ 3-B$

Q9. If $A=\{n^3+(n+1)^3+(n+z)^3;\ n\in N\}$ and $B=\{9n, n\in N\}$ then

(a) $A\subset B$
(b) $B\subset A$
(c) $A=B$
(d) $A'=B$

Q10. If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A\cup B$? Find also, the maximum number of elements in $A\cup B$.

(a) 9, 6
(b) 8, 5
(c) 7, 4
(d) 6, 3

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Answer & Solution

ANSWER

Q1	Q2	Q3	Q4	Q5
(c)	(b)	(a)	(d)	(a)
Q6	Q7	Q8	Q9	Q10
(d)	(a)	(c)	(a)	(a)